CHRISTIAN SOCIAL SERVICES COMMISSION (CSSC)

NORTHERN ZONE JOINT EXAMINATIONS SYNDICATE (NZ-JES)

FORM SIX PRE NATIONAL EXAMINATIONS 2023

141 BASIC APPLIED MATHEMATICS

MARKING SCHEME

SOLUTIONS

1.a) i) 58.699

ii) 1751.259 @02 marks

b) i) $(AB)^{-1} = \begin{pmatrix} \frac{5}{928} & \frac{57}{10672} & \frac{79}{10672} \\ \frac{13}{4176} & \frac{473}{48024} & \frac{-421}{48024} \\ \frac{-3}{928} & \frac{337}{10672} & \frac{-1}{10672} \end{pmatrix}$

ii) $|AB| = 384192$ @ 1.5 marks

c) i) $176.67^{\circ}C$

ii) 10319482.8 square metres. @ 1.5 marks

2.(a) The graph of function f,

- Intercepts:

  x-intercept; x= -4 and x= 0.5

  y-intercept; y= 2 @0.5 marks

- Asymptotes:

  Vertical asymptote; x=1 and x= -2

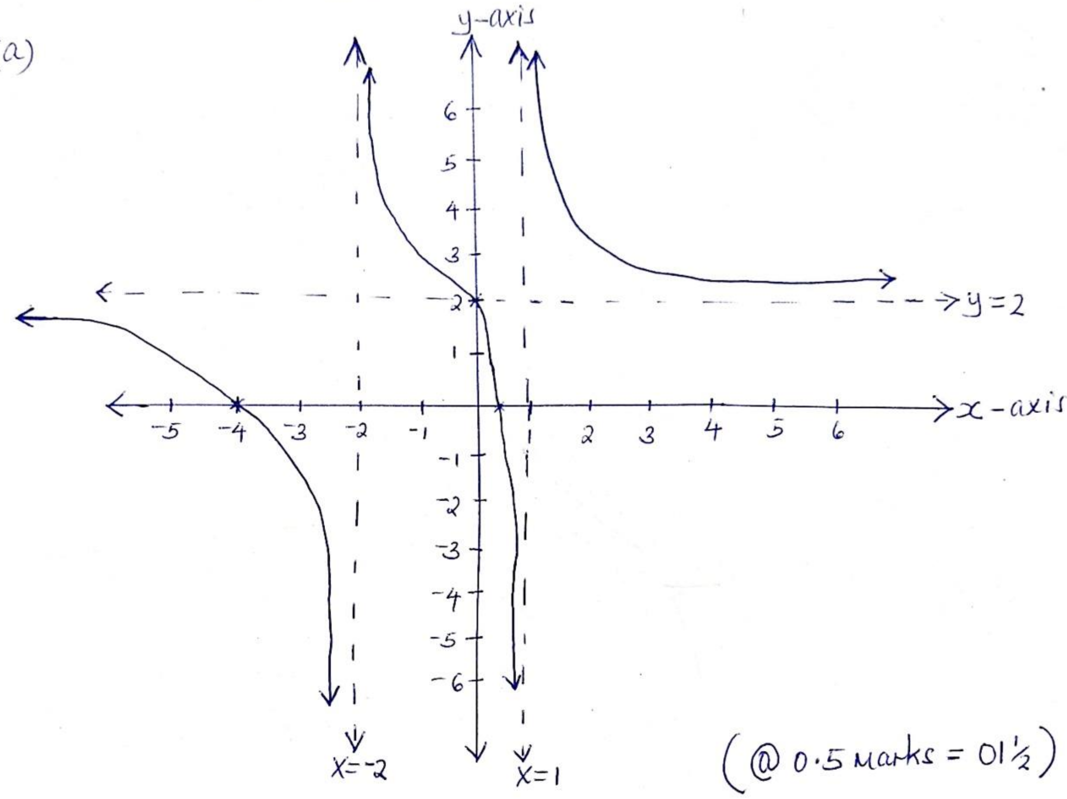
  Horizontal asymptote; y=2 @0.5 marks

- Domain = $\{x: x \in \Re,\ x \neq\ -2\ , x \neq 1\}$

  Range= $\{y: y \in \Re\}$ @0.5 marks

GRAPH OF f(x) (01/2 Marks)

2 (a)



(@ 0.5 Marks = 01½)

(b)(i)Given $f(x) = \frac{x}{x+3}$,

make x the subject,

$$yx + 3y = x,$$

$$yx - x = -3y, x(y-1) = -3y, x = -\frac{3y}{y-1}.$$

Interchange variables then, $y = -\frac{3x}{x-1}$, therfore $f^{-1}(x) = -\frac{3x}{x-1}$ (03 marks)

(ii) Given $g(x) = \frac{2}{x}$ and $f^{-1}(x) = -\frac{3x}{x-1}$ substitute $x = g(x)$ in $f^{-1}(x)$ then we have

$$f^{-1}(g(x)) = -\frac{3g(x)}{g(x)-1}, f^{-1}(g(x)) = -\frac{3\left(\frac{2}{x}\right)}{\frac{2}{x}-1} = -\frac{\frac{6}{x}}{\frac{2}{x}-1},$$

$f^{-1}(g(2/3)) = -\frac{6(2/3)}{\frac{2}{2/3}-1}, f^{-1}(g(2/3)) = \frac{-9}{2} = -4.5$ (02 marks)

3. (a) For GP, r =G4/G3= G5/G4

(d+8)/(d+3)=(d+18)/(d+8)

d^2+16d+64=d2+21d+54

d=2 (03 marks)

(b) (i) take the exponential series: 4+8+16+32+…

It is GP with G1=4 and r =2

From $G_n = G_1 r^{n-1}$ then Gn $=4(2)^{n-1} = 2^{(n+1)}$

Therefore in sigma notation is $\sum 3^{\left(2^{(n+1)}\right)}$ (02 marks)

(ii) take numerator series: 2+3+4+5+…+10

It is AP with A1=2 and d=1

Then An =A1+(n-1)d

= 2+(n-1)1

=n+1

But An=10 then n=9

Again take denominator series: 5+10+15+20+…+45

It is AP with A1=5 and d=5

Then An=5n

But An=45 then n=9

Therefore in sigma notation is

$\sum_1^9 \frac{n+1}{5n}$ (02 marks)

(c) given $\begin{cases} x+y=15 \\ x^2-y^2=75 \end{cases}$

- X=15-y
- From x^2-y^2=75
- (x-y)(x+y)=75
- X-y=5
- 15-y-y=5
- Y=5 and x=10 (03 marks)

4(a) $f(x) = \frac{1}{x^2}$

From first principle

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} \text{ ....} \mathbf{00\tfrac{1}{2}\, mark}$$

$$f'(x) = \lim_{h\to 0} \frac{\frac{1}{(x+h)^2} - \frac{1}{x^2}}{h}$$

$$f'(x) = \lim_{h\to 0} \frac{x^2-(x+h)^2}{x^2h(x+h)^2} \text{ ....} \mathbf{00\tfrac{1}{2}\, mark}$$

$$f'(x) = \lim_{h\to 0} \frac{x^2-x^2-2xh-h^2}{x^2h(x+h)^2} \text{ ....} \mathbf{00\tfrac{1}{2}\, mark}$$

$$f'(x) = \lim_{h\to 0} \frac{-2xh-h^2}{x^2h(x+h)^2} \text{ ....} \mathbf{00\tfrac{1}{2}\, mark}$$

$$f'(x) = \lim_{h\to 0} \frac{-2x-h}{x^2(x+h)^2} \text{ ....} \mathbf{00\tfrac{1}{2}\, mark}$$

as h→ 0

$$f'(x) = \frac{-2x}{x^2(x)^2} = \frac{-2x}{x^4} = \frac{-2}{x^3} \text{ ....} \mathbf{00\tfrac{1}{2}\, mark}$$

(b) $y^2 + 2xy - 3x^2 - 5 = 0$

Gradient

$$2y\frac{dy}{dx} + 2\left[x\frac{dy}{dx} + y\right] - 6x\frac{dx}{dx} = 0 \text{ ....} \mathbf{00\tfrac{1}{2} mark}$$

$$2y\frac{dy}{dx} + 2x\frac{dy}{dx} + 2y - 6x = 0 \text{ ....} \mathbf{00\tfrac{1}{2}\, mark}$$

$$(2x+2y)\frac{dy}{dx} = 6x - 2y \text{ ....} \mathbf{00\tfrac{1}{2}\, mark}$$

$$\frac{dy}{dx} = \frac{6x-2y}{2x+2y} \text{ ....} \mathbf{00\tfrac{1}{2}\, mark}$$

At (1, 2)

$$\frac{dy}{dx} = \frac{6(1)-2(2)}{2(1)+2(2)} \text{ ....} \mathbf{00\tfrac{1}{2}\, mark}$$

$$\frac{dy}{dx} = \frac{6-4}{2+4} = \frac{2}{6} = \frac{1}{3}$$

Gradient $\left(\frac{dy}{dx}\right) = \frac{1}{3}$ .... $\mathbf{00\tfrac{1}{2}\, mark}$

(c) $C_{(x)} = x^3 - 6x^2 + 9x + 15$

Marginal cost $C'_{(x)} = \frac{d(C_x)}{dx} = 3x^2 - 12x + 9$ **01mark**

At optimum cost

Marginal cost = 0 .... $\mathbf{00\tfrac{1}{2}\, mark}$

$3x^2 - 12x + 9 = 0$

$x = 1$ or $x = 3$……………………………………**01 mark**

When x=1 $C''_{(x)} = 6 - 12 = -6$ ……………………**00$\frac{1}{2}$ *mark***

When x= 3 $C''_{(x)} = 18 - 12 = 6$………………………..**00$\frac{1}{2}$ *mark***

∴ The total cost is maximum when x =1 and is minimum when x =3

…………..**00$\frac{1}{2}$ *mark***

5. (a)from the gradient function $\frac{dy}{dx} = 6x^2 - 4x$,

$dy = (6x^2 - 4x)dx$, intergrate both sides then we have $y = 2x^3 - 2x^2 + c$,

at point $p(1,3), x = 1, y = 3$

$3=2(1)^3-2(1)^2+c$

C=3 , therefore $f(x)= 2x^3-2x^2+3$ 04 marks

(b) From integral given, let $u= x^2-9$, $du/dx =2x, du/2=xdx$

Substute to the integral given then we have $u^{½} + c$, but $u = x^2 - 9$, therefore $\sqrt{x^2 - 9} + c$

(03 marks)

(c)Integral $(1 + 2\cos x)^3 \sin x dx$, rom $x = 0$, to $x = \frac{\pi}{2}$ lef $u = 1 + 2\cos x$, $du = -2\sin x\, dx$

Integrate and substute the limit ,then we have $\frac{1}{4}$ (03 marks)

6. (a) Frequency distribution table

Intervals	**Frequency**
85-89	2
90-94	3
95-99	3
100-104	5
105-109	3
110-114	4
	$\sum f = 20$

Table $\left(02\frac{1}{2}\right)$ *marks*

(b)

Intervals	f	x	d	u	fu	u^2	fu^2
85-89	2	87	-10	-2	-4	4	8
90-94	3	92	-5	-1	-3	1	3
95-99	3	97	0	0	0	0	0
100-104	5	102	5	1	5	1	5
105-109	3	107	10	2	6	4	12
110-114	4	112	15	3	12	9	36
	$\sum f = 20$				$\sum fu = 16$		$\sum fu^2 = 64$

Table $\left(02\frac{1}{2}\right)$ *marks*

For mean

$$\bar{x} = A + \frac{c\sum fu}{\sum f}$$

$$\bar{x} = 97 + 5\left(\frac{16}{20}\right)$$

$\bar{x} =$

$101g$ ....

(02)*marks*

For standard deviation

$$\text{standard deviation} = c\sqrt{\left(\frac{\sum fu^2}{\sum f}\right) - \left(\frac{\sum fu}{\sum f}\right)^2}$$

$$= 5\sqrt{\frac{64}{20} - \left(\frac{16}{20}\right)^2}$$

$= 8g$ ....(03)*marks*

7.a) (i) $\frac{\cancel{n!}}{(n-4)!} = 42\left(\frac{\cancel{n!}}{(n-2)!}\right)$

$$\frac{1}{(n-4)!} = \frac{42}{(n-2)!}$$

$$\frac{(n-2)!}{(n-4)!} = 42$$

$\frac{(n-2)\,(n-3)\,\cancel{(n-4)!}}{\cancel{(n-4)!}} = 42$ (**00½ marks**)

$n^2 - 3n - 2n + 6 = 42$

$n^2 - 5n + 6 - 42 = 0$

$n^2 - 5n - 36 = 0$

$n = 9$ or -4 (**00½ marks**)

(ii) $P(n,2) = 9P_2 = \frac{9!}{(9-2)!} = \frac{9!}{7!} = \frac{9\times8\times\cancel{7!}}{\cancel{7!}} = 72$ (**01 mark**)

$P(9,4) = \frac{9!}{(9-4)} = \frac{9}{5!} = \frac{9\times8\times7\times6\times\cancel{5!}}{\cancel{5!}} = 3024$ (**01 mark**)

a) $P(A) = {}^{3}/_{6}$, $A = \{1, 3, 5\}$ (**00½ marks**)

$P(B) = {}^{4}/_{6}$, $B = \{2, 3, 4, 5\}$ (**00½ marks**)

$A \cap B = \{3, 5\}$ (**00½ marks**)

$P(A \cap B) = {}^{2}/_{6}$ (**00½ marks**)

$P\left(\frac{A}{B}\right) = \frac{P(A\cap B)}{P(B)} = {}^{2}/_{6} \div {}^{4}/_{6}$ (**00½ marks**)

$= {}^{2}/_{\cancel{6}} \times {}^{\cancel{6}}/_{4} = {}^{2}/_{4} = {}^{1}/_{2}$

$P\left(\frac{A}{B}\right) = \frac{1}{2}$ (**00½ marks**)

b) (i) For mutually exclusive events

$P(A \cap B) = 0$

$P(A \cup B) = P(A) + P(B)$ (**01 mark**)

$= {}^{1}/_{3} + {}^{1}/_{2} = \frac{2+3}{6} = {}^{5}/_{6}$ (**01 mark**)

(ii) For non-mutually exclusive events

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$ (**01 mark**)

$= {}^{1}/_{3} + {}^{1}/_{2} - {}^{3}/_{4}$

$$= {}^{5}/_{6} - {}^{3}/_{4} = \frac{10-9}{12} = \frac{1}{12}$$

8.(a) The value of $\sin 15^{o}$

Solution:

Sin (A-B)= Sin ACosB - CosASinB (00 1/2 mark)

Let $A=45^{o}$ and $B=30^{o}$

Sin 45^{o}-30^{o}) = $\sin 45^{o}\cos 30^{o} - \cos 45^{C}\sin 30^{o}$

$= \frac{\sqrt{2}}{2} x \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} x \frac{1}{2}$ (00 1/2 mark)

Hence $\sin 15^{o} = \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}$ (00 1/2 mark)

(b) **From Cosine Rule**:

$AC^{2}=AB^{2}+BC^{2}-2AB \times BC \times \cos ABC$ (00 1/2 mark)

$=50^{2}+40^{2} -2 \times 50 \times 40 \times \cos 110^{o}$

= 2500 +1600 -4000 x (-0.3420) (00 1/2 mark)

=4100+1368

= √(5468) cm

<u>The length of AC= 73.94 cm</u> (00 1/2 mark)

(c) Prove the I dentities (i) $\sin 3\theta = 3\sin\theta - 4\sin^{3}\theta$

Use $\sin 3\theta = \sin(2\theta + \theta)$ (00 1/2 mark)

$=\sin 2\theta \cos\theta + \cos 2\theta \sin\theta$

$=\cos\theta (2\sin\theta\cos\theta)+ \sin\theta (\cos^{2}\theta-\sin^{2}\theta)$ (00 1/2 mark)

$= 2\sin\theta\cos^{2}\theta + \sin\theta (1-\sin^{2}\theta-\sin^{2}\theta)$

$=2\sin\theta(1-\sin^{2}\theta)+\sin\theta(1-2\sin^{2}\theta)$

$=2\sin\theta-2\sin^{3}\theta+\sin\theta-2\sin^{3}\theta$

<u>Hence $\sin 3\theta = 2\sin\theta-4\sin^{3}\theta$</u> (00 1/2 mark)

(ii) $(1-\cos\theta)(1+\sec\theta)=1(1+\sec\theta)-\cos\theta(1+\sec\theta)$

$=1+\sec\theta-\cos\theta-1$

$= \sec\theta - \cos\theta$ (00 1/2 mark)

$=1/\cos\theta - \cos\theta$

$(1-\cos\theta)(1+\sec\theta)=(1-\cos^{2}\theta)/\cos\theta$ (00 1/2 mark)

$=\sin^{2}\theta/\cos\theta$

= **sinθcotθ**(00 1/2 mark)

9(a) in single logarithm is:

i. $=\log_a(x^m \times \sqrt[n]{y})$ (02 marks)

ii. $=\log((x+1)^{14}(x-1)^{16})$ (02 marks)

(b) from $\log_a b = \frac{\log b}{\log a}$ then

$(\log_x y)(x\log_y Z) = \frac{\log y}{\log x} \times \frac{\log z^x}{\log y}$

$= \frac{\log Z^X}{\log y}$

$= \log_y z^x$

$= x \log_y z$ hence shown (03 marks)

(c) $y = 5^{2x}$

$\frac{dy}{dx} = \frac{d(5^{2x})}{dx}$

Let u = 2x by chain rule $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$ then

$\frac{dy}{dx} = 5^u In5 \times 2 = 2(5^{2x} In5)$

$\frac{dy}{dx} = 2(5^{2x} In5)$ (03 marks)

10.(a) Determinant of matrix A is 640

From $A^{-1} = \frac{1}{|A|} adj.A$ then

$$A^{-1} = \begin{pmatrix} \frac{-1}{8} & \frac{17}{160} & \frac{11}{160} \\ \frac{-1}{4} & \frac{1}{80} & \frac{3}{80} \\ \frac{-1}{8} & \frac{-11}{160} & \frac{7}{160} \end{pmatrix}$$ (03 marks)

(b) recall $AA^{-1} = I$ and $AI = A$ then

$$\begin{pmatrix} 2 & -6 & 2 \\ 4 & 2 & -8 \\ 12 & -14 & 16 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -2 \\ -2 \\ 14 \end{pmatrix}$$

Multiply by A^{-1} both sides

$$\begin{pmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{pmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix} == \begin{pmatrix}\frac{-1}{8} & \frac{17}{160} & \frac{11}{160}\\\frac{-1}{4} & \frac{1}{80} & \frac{3}{80}\\\frac{-1}{8} & \frac{-11}{160} & \frac{7}{160}\end{pmatrix}\begin{pmatrix}-2\\-2\\14\end{pmatrix}$$

$\therefore \begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}1\\1\\1\end{pmatrix}$ (03 marks)

(c) Decision variable

Let x be hectares of maize

Y be hectares of beans (0.5 marks)

Constraints

(i) $4x+6y\leq 26$

(ii) $1200x+800y\leq 4800$

(iii) $x\geq 0$ and $y \geq 0$ (03 marks)

Objective function

f(x, y)= x + y (0.5 mark)