CHRISTIAN SOCIAL SERVICE COMMISION (CSSC)
NORTHERN ZONE JOINT EXAMINATION SYNDICATE
FORM TWO ADDITIONAL MATHEMATICS

MARKING GUIDE:

01. (a) Given the general term $G_n = \dfrac{n^2}{2n^2-1}$, $n \rightarrow$ Natural number

at $n=1$

$$G_1 = \frac{1^2}{2(1)^2-1} = \frac{1}{2-1} = 1 \text{ ——— } \left(\tfrac{01}{2} \text{ mark}\right)$$

at $n=2$

$$G_2 = \frac{2^2}{2(2)^2-1} = \frac{4}{8-1} = \frac{4}{7} \text{ ——— } \left(\tfrac{01}{2} \text{ mark}\right)$$

at $n=3$

$$G_3 = \frac{3^2}{2(3)^2-1} = \frac{9}{18-1} = \frac{9}{17}$$

First three terms are, 1, $\frac{4}{7}$ and $\frac{9}{17}$. ——— (01 mark)

Sum of the first three term $= G_1 + G_2 + G_3$

$$= 1 + \frac{4}{7} + \frac{9}{17}$$

$$= \frac{7+4}{7} + \frac{9}{17} \text{ ——— } \left(\tfrac{01}{2} \text{ mark}\right)$$

$$= \frac{11}{7} + \frac{9}{17} = \frac{187+63}{119}$$

$$= \frac{250}{119} \text{ ——— } \left(\tfrac{01}{2} \text{ mark}\right)$$

$\Rightarrow$ Sum of first three terms is $\frac{250}{119}$.

1 (b) Next two term of the series :

(i) 7, 5, 3, 1 $\underline{-1}$ $\underline{-3}$ (Adding negative -2 to the neg.) ——— (01.5 marks)

$-2 \quad -2 \quad -2$

(ii) 2, 4, 7, 11 $\underline{16}$ $\underline{22}$ [Add the number to the first they you obtain the second number] ——— (01.5 mark)

2 3 4 5 6

(c) from pascal's triangle

n = 0 ⟶ 1.
n = 1 ⟶ 1 1
n = 2 ⟶ 1 2 1.
n = 3 ⟶ 1 3 3 1
n = 4 ⟶ 1 4 6 4 1. ——— (02 marks)
n = 5 ⟶ 1 5 10 10 5 1.
n = 6 ⟶ 1 6 15 20 15 6 1.

from our expresion: $(1+r)^6$ $n = 6$.
terms $r^0, r^1, r^2, r^3, r^4, r^5, r^6$. ——— (01 marks).
Coeficient 1, 6, 15, 20, 15, 6, 1

$$\Rightarrow (1+r)^6 = r^0 + 6r + 15r^2 + 20r^3 + 15r^4 + 6r^5 + r^6$$

$$(1+r)^6 = 1 + 6r + 15r^2 + 20r^3 + 15r^4 + 6r^5 + r^6$$ ——— (01 mark)

2 a) Given $p = \sqrt{\frac{x-1}{x+2}}$

square both side.

$\frac{p^2}{1} = \frac{x^2-1}{x+2}$ ———— (01 mark)

$\Rightarrow p^2(x+2) = x^2 - 1$

$p^2x + 2p^2 = x - 1$

$p^2x - x = -1 - 2p^2$ ———— (01 mark)

$x - p^2x = 1 + 2p^2$

$\frac{x(1-p^2)}{1-p^2} = \frac{1+2p^2}{1-p^2}$ ———— ($\frac{01}{2}$ mark)

$x = \frac{1+2p^2}{1-p^2}$. ———— ($\frac{01}{2}$ mark)

(b) let x be a number which is added to both numerator and denominator

$\Rightarrow \frac{5+x}{9+x} \times \frac{2}{3}$ ———— (01 mark)

$2(9+x) = 3(5+x)$

$18 + 2x = 15 + 3x$ ———— (01 mark)

$18 - 15 = 3x - 2x$

$3 = x \Rightarrow x = 3$ ———— (01 mark) c (i) Expand the expresion completely

$3(2c+3)^2 - c^2 = 3(2c+3)(2c+3) - c^2$ ———— $\left(\frac{01}{2}\text{ mark}\right)$

$= 3(4c^2 + 12c + 9) - c^2$ ———— $\left(\frac{01}{2}\text{ mark}\right)$

$= 12c^2 + 36c + 27 - c^2$ ———— $\left(\frac{01}{2}\text{ mark}\right)$

$3(2c+3)^2 - c^2 = 11c^2 + 36c + 27$ ———— $\left(\frac{01}{2}\text{ mark}\right)$)

(ii) $2x(x+4y) - x(8x+14y) - 2(3+4y)$

$= 2x^2 + 8xy - 8x^2 - 14xy - 6 - 8y$ ———— (01 mark)

$= -6x^2 - 6xy - 6 - 8y$. ———— $\left(\frac{01}{2}\text{ mark}\right)$

$= -6x^2 - 6xy - 6 - 8y$. ———— $\left(\frac{01}{2}\text{ mark}\right)$

3. (a) From

Sum of interior angle $= (n-2)\,180$

$2340 = (n-2)180$ ———— 02 marks

$\frac{2340}{180} + 2 = n$

$13 + 2 = n$

$15 = n$. ———— (02 marks)

The number of sides $= 15$ sides ———— (01 mark)

3 b) Given $\frac{1}{y} + y = 2\sqrt{5}$

Required to find the value of $\frac{1}{y^2} + y^2$.

From $\left(\frac{1}{y} + y\right)^2 = \left(\frac{1}{y} + y\right)\left(\frac{1}{y} + y\right)$ ———— (01 mark)

$= \frac{1}{y^2} + \frac{y}{y} + \frac{y}{y} + y^2$

$= \frac{1}{y^2} + 2 + y^2$ ———— (01 mark)

$\left(\frac{1}{y} + y\right)^2 = \frac{1}{y^2} + y^2 + 2$

$\left(\frac{1}{y} + y\right)^2 - 2 = \frac{1}{y^2} + y^2$ ———— (01 mark)

$(2\sqrt{5})^2 - 2 = \frac{1}{y^2} + y^2$

$4(\sqrt{5})^2 - 2 = y^{-2} + y^2$

$4 \times 5 - 2 = \frac{1}{y^2} + y^2$ ———— (01 mark)

$18 = \frac{1}{y^2} + y^2$

$\Rightarrow \frac{1}{y^2} + y^2 = 18$ ———— (01 mark)

4 (a)

Let $x =$ be the cube centimeter of 12.5% Concetration solution

$y =$ be the cube centimeter of 5% Concetration solution ———— ($\frac{01}{2}$ mark)

Then the volume total

$x + y = 20 \longrightarrow (i)$ ———— (01 mark)

Concetration:

$\frac{12.5\%}{100\%} x + \frac{5\%}{100\%} y = \frac{8\%}{100\%} \times 20$ ———— (01 mark) (05)

$12.5x + 5y = 8 \times 20 cm^3$

$12.5x + 5y = 160$ ———— (i)

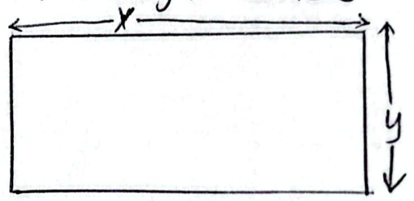
$x + y = 20$ ———— (ii) ———— (01 mark)

On solving

$x = 8 cm^3$ and $y = 12 cm^3$ ———— (01 mark)

She should mix $8 cm^3$ of 12.5% concentration solution and $12 cm^3$ of 5% concentration solution. ———— (01 mark)

4 (b) Consider rectangle below



———— (01 mark)

Total length = perimeter =

Perimeter $= 2(x+y)$

$56 cm = 2(x+y)$

$x + y = 28$ ⟶ (i) ———— (02 marks)

Area of rectangle

$A = xy$

But $A = 171 cm^2$

$171 = xy$ ⟶ (ii) ———— (01 mark)

$x = \frac{171}{y}$

Substitute value of x into (ii)

$\frac{171}{y} + y = 28$ ⟶ $171 + y^2 - 28y$

$y^2 - 28y + 171 = 0$

on solving $y_1 = 9$ $y_2 = 19$

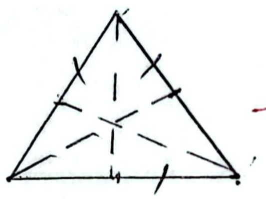
from $x = \frac{171}{y}$ when $y = 9$ $x = \frac{171}{9} = 19$ (01/2 mark)

when $y = 19$, $x = \frac{171}{19} = 9$.

$\Rightarrow$ the length of the rectangle will be $19 cm^2$ (01 mark)

The width of the rectangle will be 9cm

5 (a) order of rotation and line of symetry in an equilateral Triangle:



(03 marks)

(02 marks)

An equilateral triangle has 3 line of symetry:

(b):

Name of object	Order of rotation
(i) A rectangular playing card	2
(ii) A Ten Thousand Tanzanian Shiling	2
(iii) A nonagon	9
(iv) A pen	01
(v) A soccer ball	5

1 mark @.

06 (a) Required to show that $2x+3y-4=0$ and $3x-2y+5=0$ are perpendicular to each other:

For perpendicular $m_1 m_2 = -1$. ——— (01 mark)

from line 1 $\Rightarrow 2x+3y-4=0$.

$$3y = -2x+4$$

$$y = \frac{-2}{3}x + \frac{4}{3}.$$

$$y = mx + c$$

on comparison $m_1 = \frac{-2}{3}$. ——— (01 mark)

line 2 $\Rightarrow 3x-2y+5=0$.

$$2y = 3x+5$$

$$y = \frac{3}{2}x + 5$$

$$y = mx + c$$ ——— (01 mark)

on comparison

$$m_2 = \frac{3}{2}.$$

$$M_1 M_2 = \frac{-2}{3} \times \frac{3}{2} = \frac{-6}{6} = -1.$$

$\Rightarrow M_1 M_2 = -1$. ——— (01 mark)

Since the product of their slope is negative one hence the lines are perpendicular to each other: ——— (01 mark)

6 (b) Line 1 passes $(1,-3)$ and $(0,2)$

Line has equation $ax-3y+8=0$.

line one is parrallel to line 2:

for parallel line $m_1 = m_2$. ——— (01 mark)

slope(m) of line 1 $\Rightarrow m = \frac{\Delta y}{\Delta x}$

$$m_1 = \frac{y_2-y_1}{x_2-x_1}$$

$$M_1 = \frac{2--3}{0-1} = \frac{5}{-1}.$$

$$m_1 = -5.$$ ——— (01 mark)

from line 2 $\quad ax-3y+8=0$.

$$\frac{3y}{3} = \frac{ax+8}{3}.$$

$$y = \frac{a}{3}x + \frac{8}{3}.$$

$$y = mx + c.$$

On comparison $m_2 = \frac{a}{3}$. ——— (01 mark)

But

$$M_1 = M_2$$

$$\frac{-5}{1} = \frac{a}{3}$$

$a = -5\times 3 \Rightarrow a = -15$. ——— (01 mark)

The value of a is -15 ——— (01 mark)

7. (a) Tautology of a compound statement

Is the statement that is always true regardless of the the truth values of its component propositions. ——— (02 marks)

(b) Using truth table to determine validity of the statement;

$(p \leftrightarrow q) \wedge \sim p \wedge \sim(q \rightarrow p)$.

p	q	$\sim p$	$\overset{A}{p \leftrightarrow q}$	$q \rightarrow p$	$\overset{B}{\sim(q \rightarrow p)}$	$\overset{C}{A \wedge \sim p}$	$C \wedge B$
T	T	F	T	T	F	F	F
T	F	F	F	T	F	F	F
F	T	T	F	F	T	F	F
F	F	T	T	T	F	T	F

——— (04 marks)

Last column.

Since the last column contain all false hence the Logic statement is not valid. ——— 01

(c). The logic statement is p and q are in series with parallel of R. Then symbolically: ——— (01 marks)

$\Rightarrow (p \wedge q) \vee R$ ——— (01 marks)

logical statement will be $(p \wedge q) \vee R$. ——— (01 marks)

8. (a) Mathematicaly:

let N – be the amount of fuel used by the train

D – be the distance travelled by the train

V – be the ~~velocity~~ speed of the train

$N \propto DV^2$

$\Rightarrow N = DV^2K$ where K is constant of proportionality: (01 mrk)

$N_1 = D_1 V_1^2 K \longrightarrow (i)$

$N_2 = D_2 V_2 K \longrightarrow (ii)$

Devide eqn (ii) by (i)

$$\frac{N_2}{N_1} = \frac{D_2 V_2^2 \cancel{K}}{D_1 V_1^2 \cancel{K}}$$ (01 mrk)

$$N_2 = \left(\frac{D_2}{D_1}\right)\left(\frac{V_2}{V_1}\right)^2 N_1$$

$N_2 = ?$ $N_1 = 20L$. $D_1 = 160km$, $V_1 = 80km/hr$.

$D_2 = 320km$ · $V_2 = 40km/hr$.

$$N_2 = \left(\frac{320km}{160km}\right)\left(\frac{40 km/hr}{80 km/hr}\right) 20L$$ $\left(\frac{01}{2} \text{ mrk}\right)$

$$N_2 = \frac{2}{4} \times 20L = 10L$$

$\left(\frac{01}{2} \text{ mrk}\right)$

10 litres are used to travell 320km in 40 km/hrs.

8 b Given $d^2 \alpha \frac{h-1}{h+1}$

$\Rightarrow d^2 = \frac{k(h-1)}{(h+1)}$, where k is constant of proportionality

When $h=2$ $n=4$ $d=1$.

$1^2 = k\left(\frac{2-1}{2+1}\right)$

$\frac{1}{1} = \frac{(1)k}{3} \Rightarrow k = 3$. (01)

Equations of the expresion $d^2 = 3\left(\frac{h-1}{h+1}\right)$.

Value of h when $d=2$, $n=1$.

$2^2 = 3\left(\frac{h-1}{h+1}\right)$ (01)

$4 = \frac{3(h-1)}{h+1}$

$4h+4 = 3h-3$.

$7 = 3h-4h$

$7 = -h$

$h = -7$

$\Rightarrow$ The value of $h = -7$. (01)

8 (c) let P – be the number of people:

d – be the number of day

B – be the bill.

$P \propto \frac{B}{d}$

$P = \frac{KB}{d}$ —— (01 mrk)

$\frac{Pd}{B} = K$ (Costant of proportionality).

$\frac{P_1 d_1}{B_1} = \frac{P_2 d_2}{B_2}$ —— (01 mrk)

But $P_1 = 5$ $d_1 = 8$ days $B_1 = 120{,}000$ k

$P_2 = 6$ $d_2 = 7$ days $B_2 = ?$

$B_2 P_1 d_1 = B_1 P_2 d_2$ —— (01 mrk)

$B_2 = \frac{B_1 P_2 d_2}{P_1 d_1}$

$B_2 = \frac{120{,}000k \times 6 \times 7}{5 \times 8}$ —— ($\frac{01}{2}$ mrk)

$B_2 = 126{,}000$ k.

The bill for 6 people in 7 days is 126 000k. —— ($\frac{01}{2}$ mrk)

9. (a) Given equation

$2x - 6 \geq 4x + 3$.

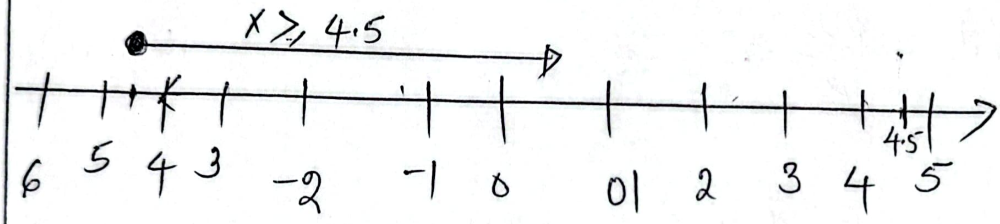
Collecting like terms

$2x - 4x \geq 6 + 3$ —————————— (01mrk)

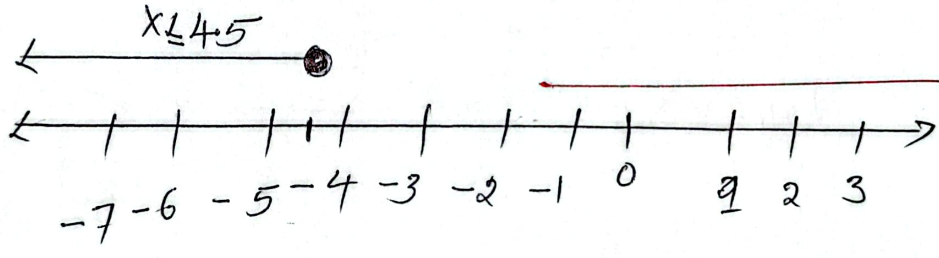
$\frac{-2x}{-2} \geq \frac{9}{-2}$

$x \cancel{\geq} -4.5$ —————————— (01 mrk)

$x \leq -4.5$.

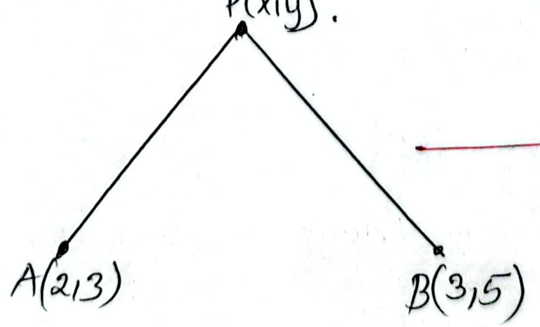


The solution $x \leq -4.5$ on number line.



—————————— (02mrks)

(b). Path described by goat.



—————————— (01mrk)

From distance formula but $\overline{PA} = \overline{PB}$ —— (01mrk)

Pg-(14)

9(b)(i) $\overline{PA} = \sqrt{(x-2)^2 + (y-3)^2}$

$\overline{PB} = \sqrt{(x-3)^2 + (y-5)^2}$. ——— ($\frac{01}{2}$ mrk)

But $\overline{PA} = \overline{PB}$

$\sqrt{(x-2)^2 + (y-3)^2} = \sqrt{(x-3)^2 + (y-5)^2}$ ——— (01 mark)

square both side.

$(x-2)^2 + (y-3)^2 = (x-3)^2 + (y-5)^2$

$x^2 - 4x + 4 + y^2 - 6y + 9 = x^2 - 6x + 9 + y^2 - 10y + 25$.

——— ($0\frac{1}{2}$ mrk)

$-4x + 6x - 6y + 10y + 13 - 34 = 0$

$2x + 4y - 21 = 0$.

The locus described by the goat is straight line ——— (01 mrks)

$2x + 4y - 21 = 0$.

10 (a) Given $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

$A = \{1, 2, 3, 4\}$

$B = \{2, 4, 6\}$

$C = \{3, 4, 5, 6\}$

Determine (i) $A' = \{5, 6, 7, 8, 9\}$ ——— (01 mrk)

(ii) $(A \cap C)'$

10 (a) $A \cap C = \{3, 4\}$.

$(A \cap C)' = \{1, 2, 5, 6, 7, 8, 9\}$ ——— (01 mrk)

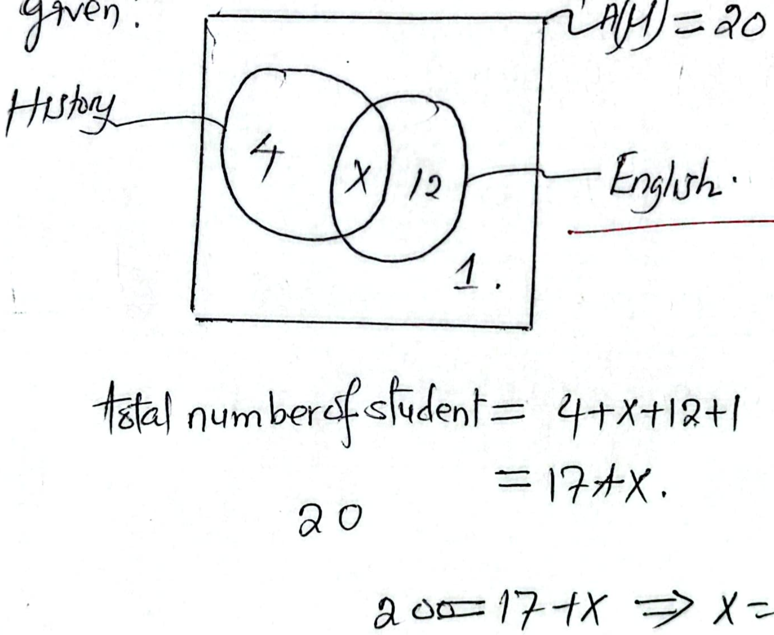
(c) $(B \cap C')'$

$C' = \{1, 2, 7, 8, 9\}$ ——— ($\frac{01}{2}$ mrk)

$(B \cap C') = \{2\}$ ——— ($\frac{01}{2}$ mrk)

$(B \cap C')' = \{1, 3, 4, 5, 6, 7, 8, 9\}$. ——— (01 mrk)

10 (b). The venn diagram to represent the information given:



——— (02 mrks)

total number of student $= 4 + x + 12 + 1$

$= 17 + x$.

20

$20 = 17 + x \Rightarrow x = 20 - 17 = 3$.

$x = 3$ ——— (01 mrks)

Number of pupil who study history = no. of student who study history only + No. of student who study history and English. ——— (01 mrk)

$= 4 + 3 = 7$ pupils ——— (01 mrk)

Therefore 7 pupils study history