PROPOSED MARKING GUIDE
ADVANCED MATHS 1

1a i) 1.149 (02)

ii) 0.084 (02)

(iii) Int[((x×Cos⁻¹(x²))/(√(2+x³))), 0.1, 0.8]

NB. IN RAD

$= 0.26325$ (02)

(10)

(b) Mean $\bar{X} = 13.935$ (02)

S.T.D $\delta = 4.474$ (02)

(10)

2(a) $\tan(\ln x) = \frac{x^2-1}{x^2+1}$

$= \frac{x^2-1}{x^2+1}$

LHS.

$\tan(\ln x) = \frac{\sin(\ln x)}{\cos(\ln x)}$

$= \frac{e^{2\ln x}-1}{e^{2\ln x}+1}$ (01)

(10)

$= \frac{e^{\ln x^2}-1}{e^{\ln x^2}+1}$ (01)

But

$e^{\ln x^2} = x^2$

$= \frac{x^2-1}{x^2+1}$ (01)

shown.

(b) $2\cosh 2x + 10\sinh 2x = 5$

$2\left[\frac{e^{2x}+\frac{1}{e^{2x}}}{}\right] + 10\left[e^{2x}-\frac{1}{e^{2x}}\right] = 5$

$\Rightarrow 2\left[\frac{e^{4x}+1}{e^{2x}}\right] + 10\left[\frac{e^{4x}-1}{e^{2x}}\right] = 5$ (00½)

$\Rightarrow 2e^{4x}+2+10e^{4x}-10 = 5e^{2x}$

$\Rightarrow 12e^{4x}-5e^{2x}-8 = 0$ (00½)

Let $e^{2x} = P$

$\Rightarrow 12P^2-5P-8=0$

$\Rightarrow 6P^2-5P-4=0$

$P = \frac{5 \pm \sqrt{121}}{12}$ (00½)

(b) $P = \frac{5 \pm 11}{12}$

$P = \frac{-6}{12}$ or $\frac{16}{12}$

But $P = e^{2x}$

$e^{2x} = \frac{-6}{12}$ or $\frac{16}{12}$ (00½)

$2x = \ln(-\frac{1}{2})$ or $\ln \frac{16}{12}$

$\ln(-\frac{1}{2})$ N.A.

Thus

$2x = \ln \frac{16}{12}$ (00½)

$= \ln \frac{4}{3}$

$x = \frac{1}{2}\ln(\frac{4}{3})$

OR

$x = \ln(\frac{2}{\sqrt{3}})$ (00½)

OR

$x = \ln(\frac{2\sqrt{3}}{3})$

OR

$x = 0.1438$

(c) $\int_4^7 \frac{dx}{\sqrt{4x^2-8x-5}}$

By completing squares

$4x^2-8x-5 = 4(x^2-2x)-5$

$= [2(x-1)]^2 - 9$

$= 9\left[\left[\frac{2}{3}(x-1)\right]^2 - 1\right]$ (01)

By substitution.

$\int_4^7 \frac{dx}{\sqrt{9\left[\left[\frac{2(x-1)}{3}\right]^2 - 1\right]}}$

$\int_4^7 \frac{dx}{3\sqrt{\left[\frac{2(x-1)}{3}\right]^2 - 1}}$ (01)

Let

$\frac{2}{3}(x-1) = \cosh y$

$dx = \frac{3}{2}\sinh y\,dy$

$\int_a^b \frac{\frac{3}{2}\sinh y\,dy}{3\sqrt{\cosh^2 y - 1}}$ (01)

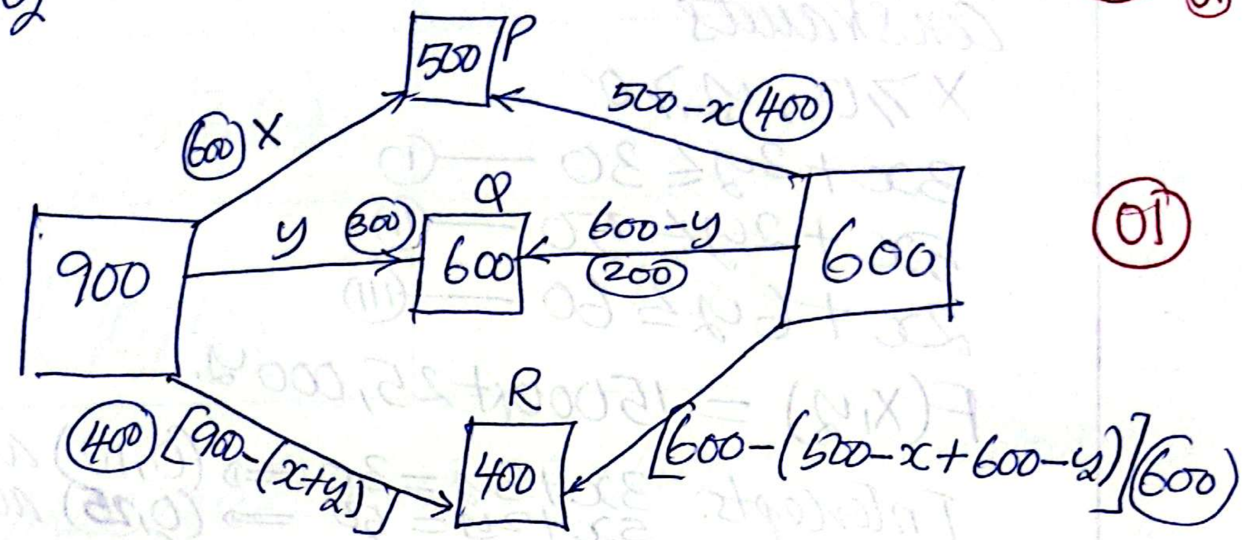
$\int_a^b \frac{1}{2}\frac{\sinh y\,dy}{\sinh y}$

$\frac{1}{2}\int_a^b dy = \frac{1}{2}(y)_a^b$

$= \frac{1}{2}\cosh^{-1}\left[\frac{2}{3}(x-1)\right]_4^7$ (01)

(02) $= \frac{1}{2}[0.74648] \simeq 0.3732$

3(a) Let x be number of bricks transported from H_1 to P
y " " H_1 to Q (01)



(01)

Constraints

$x \geq 0, \; y \geq 0$

$x + y \leq 900$ ——— (i)

$x \leq 500$ ——— (ii)

$y \leq 600$ ——— (iii)

$x + y \geq 500$ ——— (v)

(01)

$F(x,y) = 600x + 300y + 400[900-(x+y)] + 400[500-x] + 600(600-y) + 600[x+y-500]$

$= 600x + 300y + 360000 - 400x - 400y + 200000 - 400x + 360000 - 600y + 600x + 600y - 300000$

$= 400x - 100y + 620,000$ (01)

3(b) Let x be the number of chairs to be made (01)
y " Tables to be made

Constraints

$x \geq 0$, $y \geq 0$

$3x + 3y \leq 30$ —— (I)

$5x + 2y \leq 50$ —— (II)

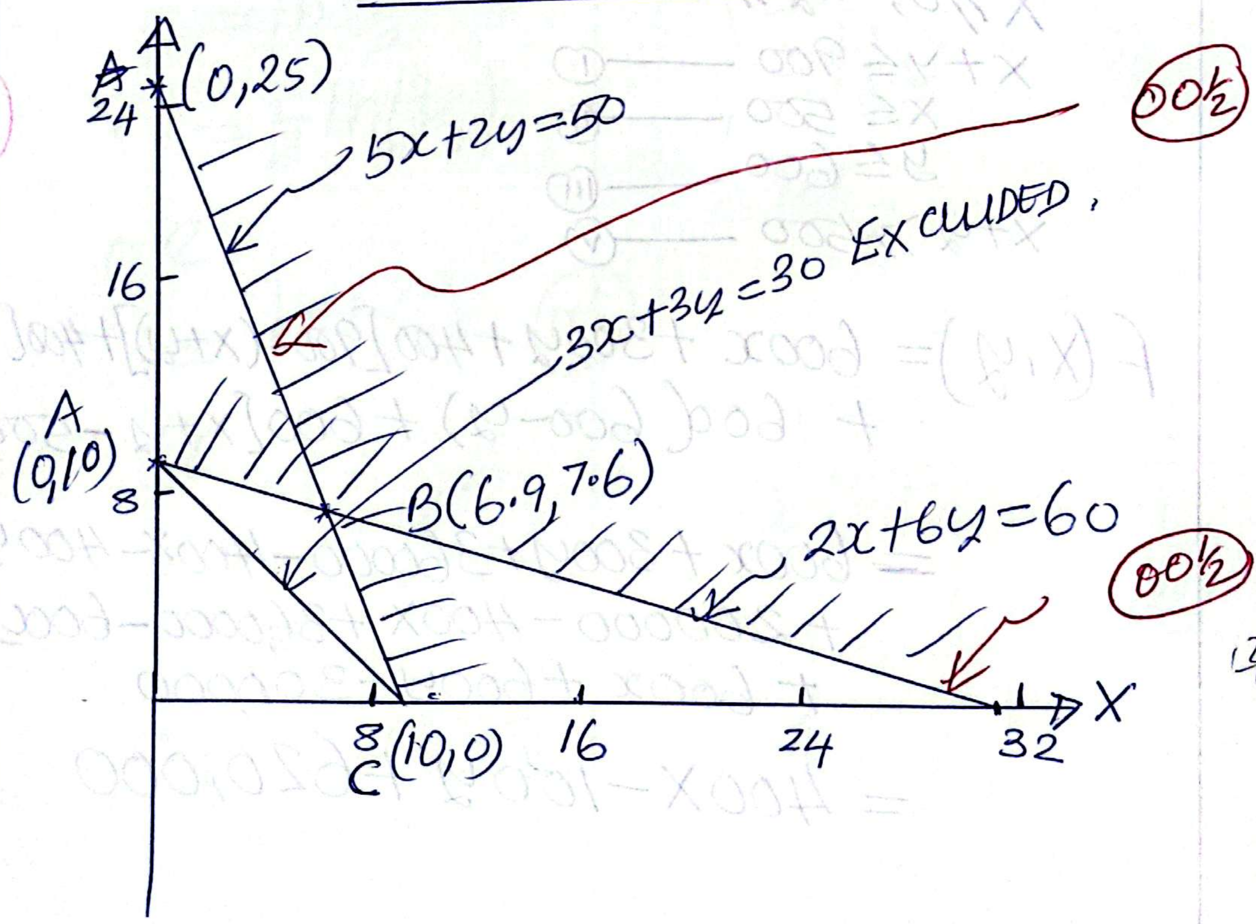
$2x + 6y \leq 60$ —— (III)

(01)

$F(x, y) = 15000x + 25,000y.$

Intercepts: $3x + 3y = 30 \Rightarrow (0,10)$ and $(10,0)$
$5x + 2y = 50 \Rightarrow (0,25)$ and $(10,0)$
$2x + 6y = 60 \Rightarrow (0,10)$ and $(30,0)$

GRAPH. SKETCH.



CORNER POINTS.	$F(x,y) = 15000x + 25,000y$
A(0,10)	250,000/= (00½)
~~B(6.9,7.6)~~ B (6,7)	~~293,500/=~~ 265,000
C(10,0)	150,000/= (00½)

Machine A is Excluded in production
For Maximum income: $B(x,y) = 6.9, 7.6 \simeq (6,7)$
6 chairs and 7 table for a month.

(i) For two years: $(6,7) \times 24$ Months (01)
There 144 chairs and 168 tables.

(ii) ~~$F(x,y) \times \frac{3}{12}$~~ Quarter of a year $= \frac{12}{4} = 3$ months

Maximum income $\Rightarrow F(x,y) \times 4 = [15000x + 25000y]4$
$\Rightarrow F(6,7) \times 4 = 1060,000/=$
(01)

⟨05⟩

Given

4(a) 3, 16, 6, 15, 11, 6, 8, 9.

⇒ Ascending order

~~3, 6, 8, 9~~

⇒ 3, 6, 6, 8, 9, 11, 15, 16

(i) $Median = \frac{\left(\frac{N}{2}\right)^{th} + \left(\frac{N}{2}+1\right)^{th}}{2}$ (00½)

N = 8, No of obs.

$= \frac{\left(\frac{8}{2}\right)^{th} + \left(\frac{8}{2}+1\right)^{th}}{2}$ (00½)

$= \frac{4^{th} + 5^{th}}{2}$

The 4^{th} term = 8

The 5^{th} term = 9.

∴ $Median = \frac{8+9}{2}$

$= 8.5$ (00½)

(05)

4(a)(ii)

$Variance = \frac{\Sigma f(x-\bar{x})^2}{\Sigma f}$ (00½)

$= \frac{\Sigma f(x-\bar{x})^2}{N}$.

$\bar{X} = Mean = \frac{\Sigma x}{N}$

$= \frac{74}{8}$

$= 9.25$

x	f	$x-\bar{x}$	$= f(x-\bar{x})^2$
3	1		39.0625
6	2		21.125
8	1		1.5625
9	1		0.0625
11	1		3.0625
15	1		33.0625
16	1		45.5625
			143.5

(00½)

$Variance = \frac{\Sigma f(x-\bar{x})^2}{N}$

$= \frac{143.5}{8}$ (00½)

$= 17.9375$

∴ $Variance = 17.9375$

(00½)

(06?) ...

4(b) $\sqrt{\frac{\sum f(x-\bar{x})^2}{\sum f}} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$

LHS.

Let $\bar{x} = \mu$ = Mean.

$$\sqrt{\frac{\sum f(x-\mu)^2}{\sum f}}$$

$$\sqrt{\frac{\sum f(x^2-2x\mu+\mu^2)}{\sum f}} \quad \text{-----} \quad 00\tfrac{1}{2}$$

$$\sqrt{\frac{\sum fx^2}{\sum f} - 2\mu\frac{\sum fx}{\sum f} + \mu^2}, \quad \text{But } \frac{\sum f \cdot \mu^2}{\sum f} = \mu^2 \quad 00\tfrac{1}{2}$$

and $\mu^2 = \left(\frac{\sum fx}{\sum f}\right)^2$

$$\sqrt{\frac{\sum fx^2}{\sum f} - 2\left(\frac{\sum fx}{\sum f}\right)\left(\frac{\sum fx}{\sum f}\right) + \left(\frac{\sum fx}{\sum f}\right)^2} \quad \text{-----} \quad 00\tfrac{1}{2}$$

$$\sqrt{\frac{\sum fx^2}{\sum f} - 2\left(\frac{\sum fx}{\sum f}\right)^2 + \left(\frac{\sum fx}{\sum f}\right)^2} \quad \text{-----} \quad 00\tfrac{1}{2}$$

$$\sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2} \quad \text{hence Proved}$$

RHS = LHS $00\tfrac{1}{2}$

4C

KM	f	C.mark (X)	d	$u=\frac{d}{c}$	fu	fu^2
0-4	45	2	-5	-1	-45	45
5-9	58	7	0	0	0	0
10-14	27	12	5	1	27	27
15-19	30	17	10	2	60	120
20-24	19	22	15	3	57	171
25-29	11	27	20	4	44	176
30-34	8	32	25	5	40	200
35-39	2	37	30	6	12	72
					195	811
	200				$00\frac{1}{2}$	$00\frac{1}{2}$

$A=7, C=5, d=X-A$

(i) Mean $\bar{X} = A + C\frac{\Sigma fu}{\Sigma f}$ ---- $00\frac{1}{2}$

$= 7 + 5\frac{(195)}{200}$ ---- $00\frac{1}{2}$

$\bar{X} = 11.875$ ---- $00\frac{1}{2}$

(ii) Standard deviation δ.

$\delta = c\sqrt{\frac{\Sigma fu^2}{\Sigma f} - \left(\frac{\Sigma fu}{\Sigma f}\right)^2}$ ---- $00\frac{1}{2}$

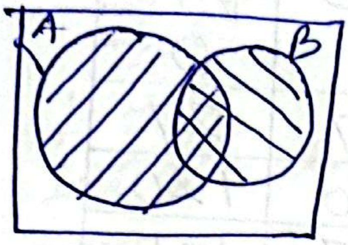
$= 5\sqrt{\frac{811}{200} - \left(\frac{195}{200}\right)^2}$ ---- $00\frac{1}{2}$

$= 5\sqrt{1.76192}$

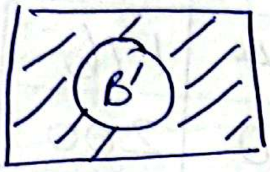
$= 8.81$

Standard deviation $\delta = 8.81$ $00\frac{1}{2}$

5(a) $(A \cup B) - B = (A \cup B) \cap B'$



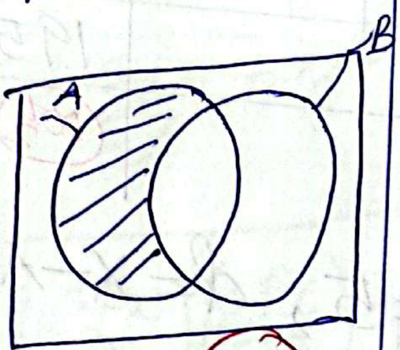
$= A \cup B$



$= B'$

$n(A \cup B) \cap B' \in A \cap B'$

$(A \cup B) \cap B' =$

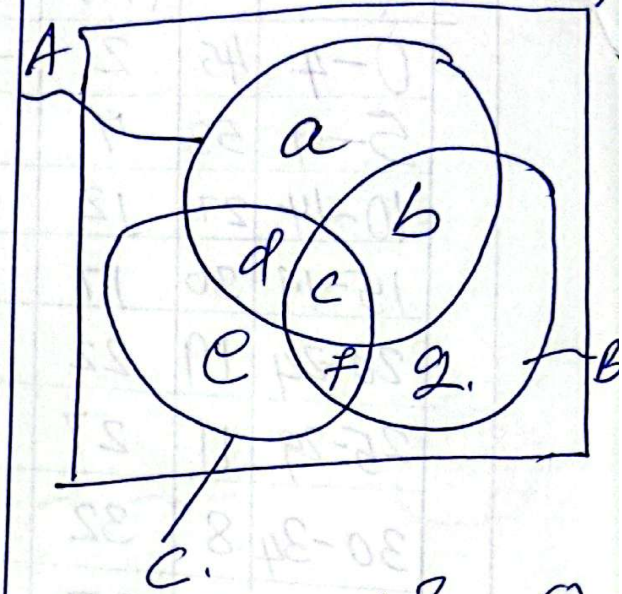


(ii) $(A - B) \cup C$

$(A - B) = A \cap B'$

$(A \cap B') \cup C$

Consider elements
a, b, c, d, e, f, g



$A = \{a, b, c, d\}$ — (i)

$B = \{b, c, f, g\}$

$C = \{d, c, f, e\}$

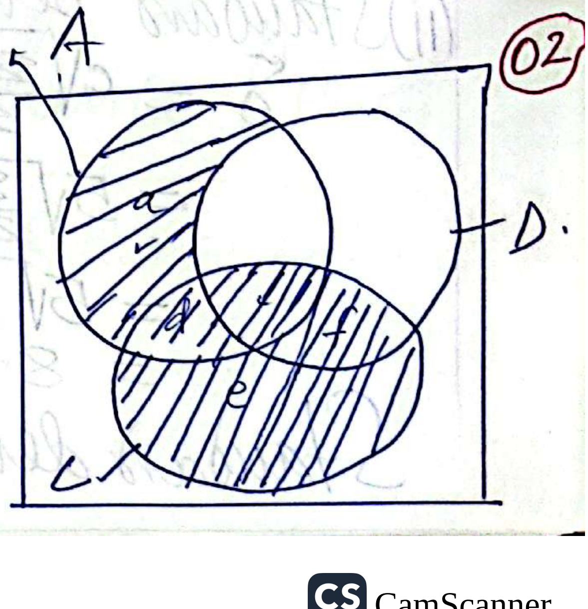
$B' = \{a, d, e\}$ — (ii)

$\therefore A \cap B' = \{a, d\}$

$A \cap B' = a, d,$

$(A \cap B') \cup C = \{a, d, c, f, e\}$

SOLUTION



5(b) ~~n B+I~~

$n(A \cup B \cup C) = 23$

$n(A \cup C) = 18$

$n(B) = 7$

$n[(A \cup C) \cup B] = n[A \cup B \cup C]$

Let $A \cup C = D$ - given

$n(D \cup B) = n(A \cup C) \cup B$

Define

(10) $n(D \cup B) = nD + nB - n(D \cap B)$

$n(D \cup B) = nD + nB - n(D \cap B)$

By Substitution

(01) $23 = 18 + 7 - n(D \cap B)$

$n(D \cap B) = 25 - 23$

$= 2$

$\therefore n(D \cap B) = 2$ —— (1)

(10) $n(A \cup C) - B = n(A \cup C) \cap B'$

$n(A \cup C) - B = n(D \cap B')$ —— (ii)

$n(A \cup C) - B$

From.

$nD = n(D \cap B) + n(D \cap B')$

$n(A \cup C) = n(D \cap B) + n(D \cap B')$

(01) $18 = 2 + n(D \cap B')$

$\therefore n(D \cap B') = 18 - 2$

$= 16$

$\therefore n[A \cup C] - B = 16$ (01)

(c) $n(P) = 12$ P - physics

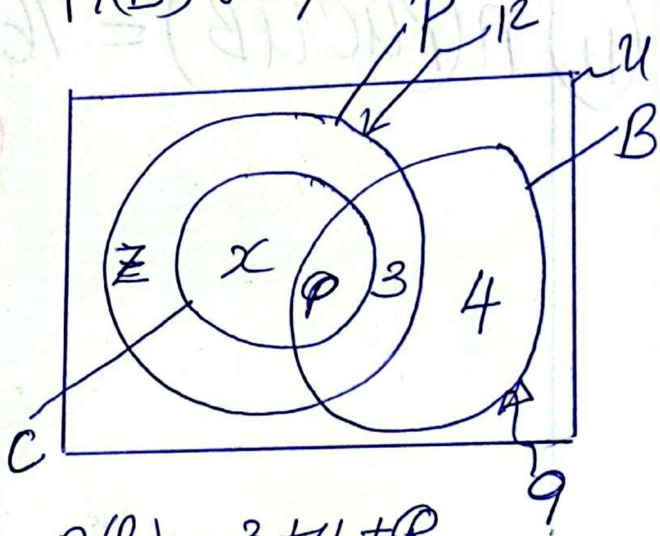
$n(B) = 9$ B - Biology

$n(C) = 3$ C - Chemistry

$n(C \in P)$.

$n(P \cap C \cap B) =$

$n(B)$ only $= 4$



$n(B) = 3 + 4 + \varphi$

$9 = 7 + \varphi$

$\varphi = 2$ (00½)

But chemistry $= 3$

$x + \varphi = 3$

$x + 2 = 3$

$x = 1$

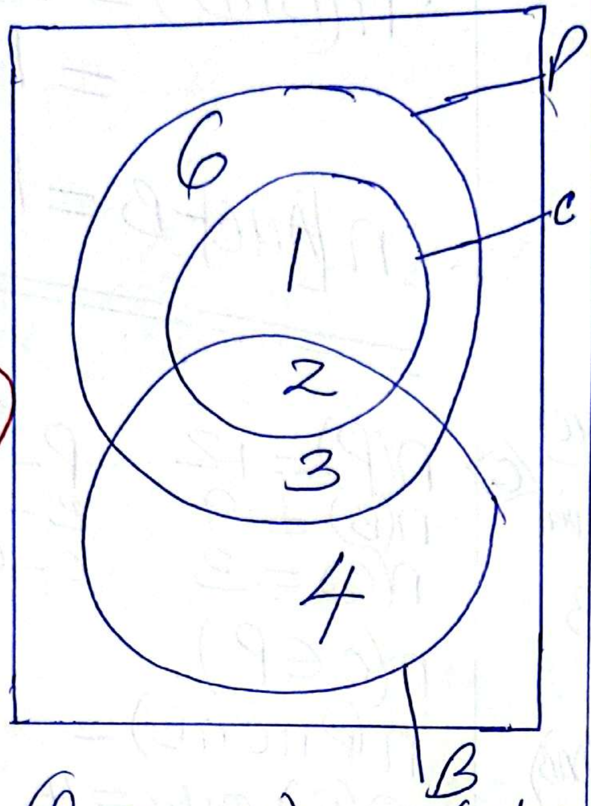
$nP = \varphi + x + 3 + z$

$12 = 2 + 1 + 3 + z$

$z = 6$ (00½)

5(c) VENN DIAGRAM

(i)



02

(ii) $n(P \cup C \cup B) = 16$ students.

01

6 (a) First programed function

$B(x) = 2x^2 - 7x$

2nd sub function operating

$H(x) = 3x - 5$

By composite function

Subfunction is the domain of First function

$B \circ H(x) \Rightarrow B(3x-5)$

But $B(x) = 2x^2 - 7x$.

$B \circ H(x) = 2(3x-5)^2 - 7(3x-5)$

01

$= 18x^2 - 6x + 50$
$- 21x + 35$

$= 18x^2 - 81x + 85$

∴ New operating function

01

(i) $B \circ (H(x)) = 18x^2 - 81x + 85$

6(b)(i) $B(H(x)) = 18x^2 - 81x + 85$

$B'(x) = 36x - 81$ - - - - - - - 01

for max. or minimum values $B'(x) = 0$.

$36x = 81$

$x = 81/36 \simeq$

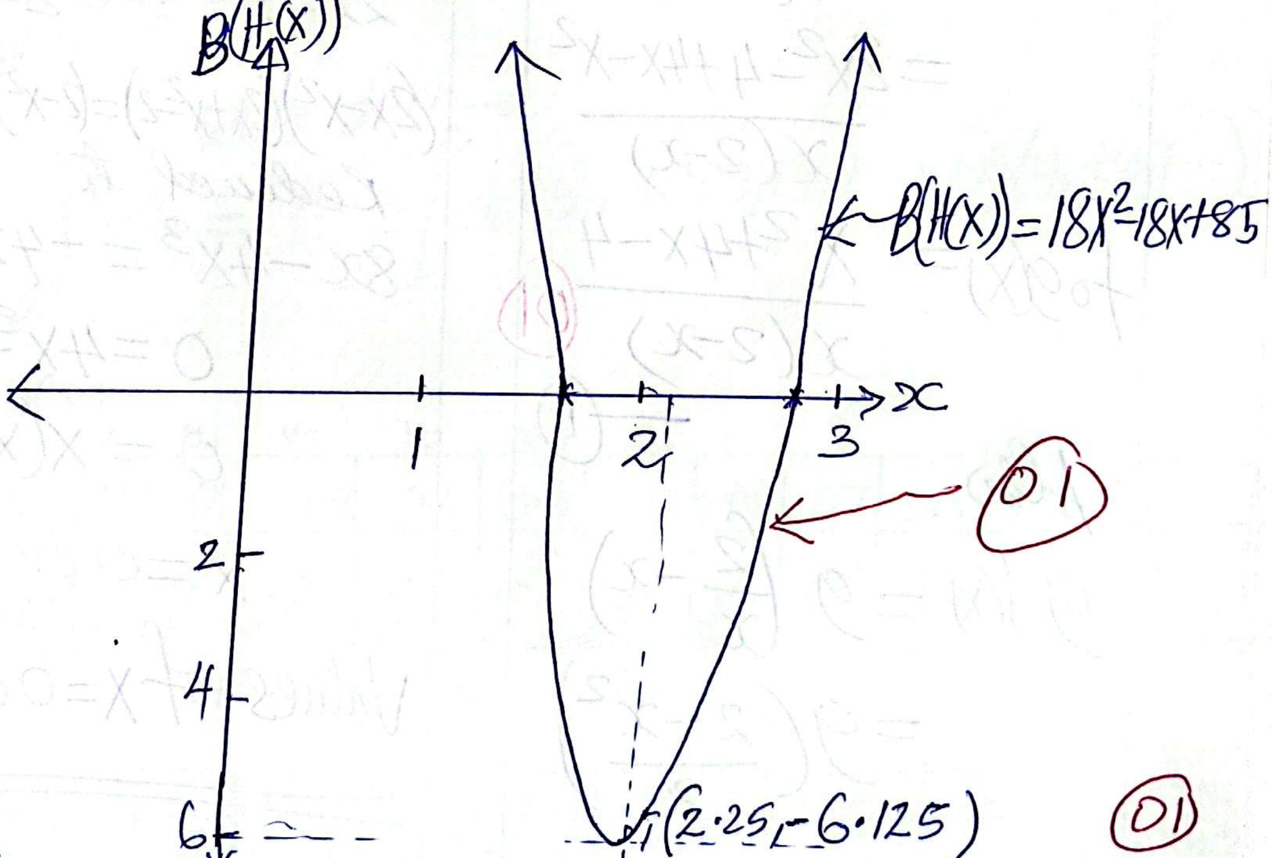
$BH(x) = 18(81/36)^2 - 81(81/36) + 85$ - - - - - - 01

$= -6\frac{1}{8}$

Intercepts: $y = 0$, $x_1 = 2.83$ or $x_2 = 1.66$

$B_H''(x) = 36 \Rightarrow B''H(x) \geq 0 \Rightarrow$ minimum

SKETCH



01

(ii) The minimum value of $B[H(x)] = -6\frac{1}{8}$ units

6(c) $f(x) = \frac{2}{x} - x$

$g(x) = \frac{2-x}{x}$

$fg(x) = f\left(\frac{2-x}{x}\right)$

$= \frac{2x}{2-x} - \left(\frac{2-x}{x}\right)$

$= \frac{2x^2 - (2-x)(2-x)}{(2-x)x}$

$= \frac{2x^2 - [4-4x+x^2]}{x(2-x)}$

$= \frac{2x^2 - 4 + 4x - x^2}{x(2-x)}$

$f\circ g(x) = \frac{x^2+4x-4}{x(2-x)}$ (01) — (1)

Also.

$g f(x) = g\left(\frac{2}{x} - x\right)$

$= g\left(\frac{2-x^2}{x}\right)$

$g f(x) = \frac{2 - \left(\frac{2-x^2}{x}\right)}{\left(\frac{2-x^2}{x}\right)}$

$= \frac{2x - (2-x^2)}{x} \Big/ \frac{2-x^2}{x}$ (01)

$= \frac{2x + x^2 - 2}{2-x^2}$ — (11)

But $g f(x) = 1$

$f\circ g(x) = 1$

Equating

$\frac{x^2+4x-4}{2x-x^2} = \frac{2x+x^2-2}{2-x^2}$ (01)

$(2x-x^2)(2x+x^2-2) = (2-x^2)(x^2+4x-4)$

Reduced to

$8x - 4x^3 = -4x$

$0 = 4x^3 - 12x$

$0 = x(x^2-3)$

$x = 0$ or $x = \pm\sqrt{3}$

Values of $x = 0$ or $\pm\sqrt{3}$ (01)

7(a)

$$\int_{-d}^{d} y\,dx = \frac{2}{3}ad^3 + 2cd, \quad \text{But } c = y_1$$

$$= \frac{2}{3}ad^3 + 2y_1 d$$

By substituting $a = \frac{1}{2d^2}(y_0 + y_2 - 2y_1)$.

$$= \frac{2}{3}d^2\left[\frac{1}{2d^2}(y_0 + y_2 - 2y_1)\right] + 2y_1 d$$

$$= \frac{1}{3}d(y_0 + y_2 + 4y_1)$$

For 'n' strips

$$\int_{-d}^{d} y\,dx = \frac{1}{3}\left[d\left[(y_0 + y_2 + 4y_1) + (y_2 + y_4 + 4y_3) + \dots + y_n\right]\right]$$

$$= \frac{d}{3}\left(y_0 + y_n + 2(y_2 + y_4 \dots) + 4(y_1 + y_3 + \dots)\right)$$

$\Rightarrow$ OR

$$\int_{-d}^{d} y\,dx = \frac{d}{3}\left[\text{Sum of 1st + last ordinate} + 2\sum \text{even ord} + 4\sum \text{odd ordin}\right.$$

where $d = b - a$.

$$\therefore \int_{a}^{b} y\,dx = \frac{b-a}{3}\left[y_0 + y_n + 2(y_2 + y_4 + \dots) + 4(y_1 + y_3 + \dots)\right]$$

7(b) Given

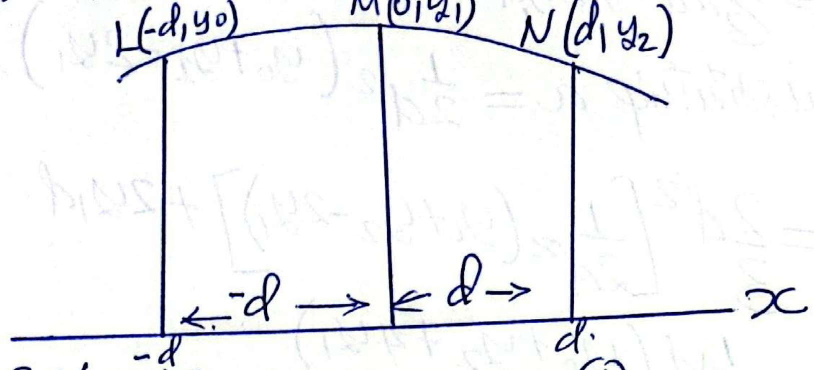
	y_0	y_1	y_2	y_3	y_4	y_5					
X	0	0.1	0.2	0.3	0.4	0.5	.06	0.7	0.8	0.9	1.0
f(x)	1	0.90909	0.8333	0.76903	0.71409	0.666	0.625	0.5882	0.5536	0.5263	0.5

for $h = 0.2$

	y_0	y_1	y_2	y_3	y_4	y_5
X	0	0.2	0.4	0.6	0.8	1
f(x)	1	~~0.90909~~ 0.8333	0.71409	0.625	0.5536	0.5

(01)

7(a) Consider the curve. $Y = ax + by + c$



$Y = ax^2 + bx + c$ ——— (I)

By substituting y_0, y_1, y_2 in (I):

$y_0 = ad^2 - bd + c$ ——— (I)

$y_2 = ad^2 + bd + c$ ——— (II)

$y_1 = c$ ——— (III)

~~$y_2 + y_1 = 2ad^2 + bd$~~ By eliminating constant a, b, c.

Adding (I) and (II)

$y_0 + y_2 = 2ad^2 + 2c$

But $c = y_1$

$y_0 + y_2 = 2ad^2 + 2y_1 \Rightarrow y_0 + y_2 - 2y_1 = 2ad^2$

$a = \frac{1}{2d^2}(y_0 + y_2 - 2y_1)$ ——— (IV)

The area enclosed btn $-d$ to d

$A = \int_{-d}^{d} y\,dx$

$= \int_{-d}^{d} (ax^2 + bx + c)\,dx$

$= \left[\frac{ax^3}{3} + \frac{b}{2}x^2 + cx\right]_{-d}^{d}$

$= \frac{2}{3}ad^3 + 2cd$

CONTINUE:

TRAPEZOIDAL RULE

7 $\int_a^b y\,dx = \frac{h}{2}\left[\Sigma(1^{st}\text{ ordinate} + \text{last ordinate}) + 2(\Sigma(\text{Rest ordinate}))\right]$

$= \frac{b-a}{2}\left[\Sigma(y_0 + y_n + 2(y_1 + y_2 + \cdots))\right]$. (00½)

$\Rightarrow$ But $b-a = h = 0.2$. (00½)

$= \frac{0.2}{2}\left(1 + 0.5 + 2(0.8333 + 0.71409 + 0.625 + 0.5536)\right)$

$= 01\ (1.5 + 5.45198)$

$A' = 0.695198$ ——— (1) (00½)

FOR SIMPSON'S RULE.

$\int_a^b y\,dx = \frac{0.2}{3}\left[1.5 + 4(0.8333 + 0.625) + 2(0.71409 + 0.5536)\right]$ (00½)

$= \frac{0.2}{3}[9.86858]$.

$= 0.65790533$

$A \simeq 0.657905$ ——— (1) (00½)

(c) Error $(\Delta A) = |A - A'|$ (01)

$= |0.657905 - 0.695198|$

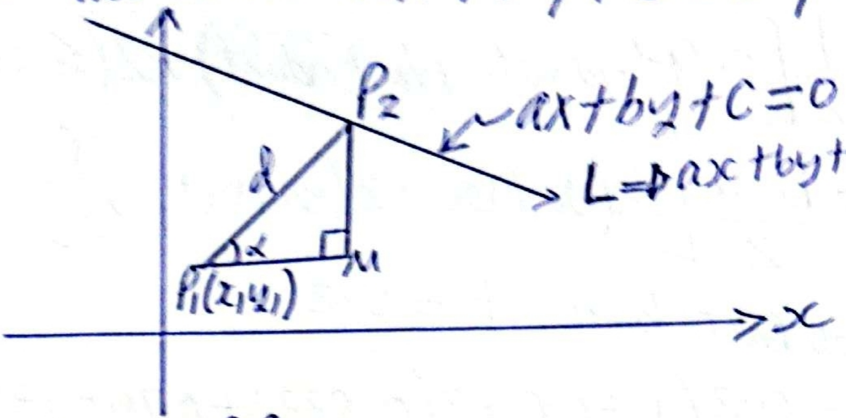
$= |-0.0371|$

% error $= \left|\frac{A - A'}{A}\right| \times 100\%$ (01)

$= \frac{0.037293}{0.657905}$

The percentage error $= 5.67\%$ (01)

8(a) Consider the line $ax+by+c=0$ passing at Point P_2



(00½)

Consider ΔP_1P_2M

$\sin\alpha = \frac{P_2M}{d} \iff P_2M = d\sin\alpha$

$\cos\alpha = \frac{MP_1}{d} \iff P_1M = d\cos\alpha$ (00½)

Then at $P_2(x_1+d\cos\alpha,\ y_1+d\sin\alpha)$

Since P_2 lies on $ax+by+c=0$

Thus it must satisfy $a(x_1+d\cos\alpha)+b(y_1+d\sin\alpha)+c=0$

$ad\cos\alpha + bd\sin\alpha + ax_1 + by_1 + c = 0$.

$$d = -\frac{(ax_1+by_1+c)}{a\cos\alpha + b\sin\alpha} \quad \text{—— (I)}$$

(00½)

Assume P_1P_2 is perpendicular to $L \Rightarrow ax+by+c=0$

slope $(M_1) = -a/b$

Then the Slope $P_1P_2 = \frac{b}{a}$, From $M_1M_2 = -1$.

But $\tan\alpha = b/a = M_2$

$\tan^2\alpha = b^2/a^2$ From relation of Identity Trig.

$\tan^2\alpha + 1 = \sec^2\alpha$.

$\sec\alpha = \sqrt{1+\tan^2\alpha} = \sqrt{1+\frac{b^2}{a^2}} = \sqrt{\frac{a^2+b^2}{a^2}}$ (00½)

$\therefore \cos\alpha = \pm\frac{a}{\sqrt{a^2+b^2}}$ —— (II)

Also $\tan\alpha = \frac{\sin\alpha}{\cos\alpha}$

$$\sin\alpha = \cos\alpha \tan\alpha$$

$$= \left(\frac{a}{\sqrt{a^2+b^2}}\right)\frac{b}{a}$$

$$\sin\alpha = \frac{b}{\sqrt{a^2+b^2}} \quad \text{--- (iii)}$$

(00½)

By Substituting (ii) and (iii) in (i).

$$d = \frac{-(ax_1+by_1+c)}{a\left(\frac{a}{\sqrt{a^2+b^2}}\right)+b\left(\frac{b}{\sqrt{a^2+b^2}}\right)}$$

(00½)

$$= \frac{-(ax_1+by_1+c)}{\frac{1}{\sqrt{a^2+b^2}}(a^2+b^2)}$$

$$= \frac{-(ax_1+by_1+c)}{\frac{\sqrt{a^2+b^2}}{a^2+b^2}(a^2+b^2)}$$

(00½)

~~= -a(~~

$$d = \frac{-(ax_1+by_1+c)}{\sqrt{a^2+b^2}}$$

(0½)

∴ The Perpendicular distance 'd' $= \left|\frac{-(ax_1+by_1+c)}{\sqrt{a^2+b^2}}\right|$

(10)

8(b) If $(x_1, y_1) = 3,2$

$\frac{3}{4}x - y = -1.$

$\Rightarrow 3x - 4y = -4$ ——— 01

$\Rightarrow 3x - 4y + 4 = 0$

$d = \left| \frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}} \right|$ ——— (01)

$= \pm \left| \frac{3(3) + -4(2) + 4}{3^2 + (-4)^2} \right|$

$= \pm\ 5/5$ ——— (01)

C $C_1: 2x^2 - 2y^2 - 3x - 9y + 2 = 0$ — (i)

$C_2: 2x^2 + 2y^2 + 12x - 6y + 8 = 0$ — (ii) or $x^2 + y^2 + 6x - 3y + 4 = 0$

point of insection

Equating $i = ii$

$15x + 3y + 6 = 0 \Rightarrow 5x + y + 2 = 0$ (01)

$\Rightarrow y = -5x - 2$ —— (iii)

Substituting (iii) in (i) or (ii) for solving x, y.

$C_2: x^2 + y^2 + 6x - 3y + 4 = 0$

$x(-5x-2)^2 + 6x - 3(-5y-2) + 4 = 0$

$26x^2 + 41x + 14 = 0$

$x = -\frac{1}{2}$ or $\frac{14}{13}$. , For $x = -\frac{1}{2}$, $y = \frac{1}{2}$. (01)

$x = \frac{14}{13}$, $y = -\frac{96}{13}$

slope $\overline{PQ} = \frac{-\frac{96}{13} - \frac{1}{2}}{\frac{14}{13} + \frac{1}{2}} = 4\frac{15}{41}$

The Gradient $= 4\frac{15}{41}$ (01)

9 (a) (iii) $\int_0^{\pi/2} \frac{\cos\sqrt{x}}{\sqrt{x}}$

Let $u=\sqrt{x} \iff u^2=x$.

$2u\,du=dx$. (01)

$\Rightarrow \int \frac{\cos u}{u}.2u\,du$

$\Rightarrow 2\int \cos u\,du$

$= 2\sin u\Big|_a^b$

$= 2\sin\sqrt{x}\Big|_0^{\pi/2}$

$= 0.3294$

$= 0.329$. (01)

(b) $\int 3^{\sqrt{2x+1}}$

Let $a=\sqrt{2x+1}$,

$a^2=2x+1$

$2a\,da=2dx$

$dx=a\,da$. (00½)

$\int 3^{\sqrt{2x+1}}dx=\int 3^a a\,da$

Let $u=a$ and $dv=3^a da$

$du=da$ $\quad v=\int 3^a da$

$v=\frac{3^a}{\ln 3}$. (00½)

(b) Let $b=3^a$ —— (i)

$\ln b=a\ln 3$. (00½)

$\frac{d}{da}(\ln b)=\ln 3\,da$.

$\frac{db}{b}=\ln 3\,da$.

$da=\frac{1}{b\ln 3}db$. —— (ii)

Then.

$V=3^a.da$.

$\Rightarrow$ By substituting (i) (ii):

$=b.\frac{1}{b\ln 3}db$.

$V=\frac{1}{\ln 3}db$

$=\frac{b}{\ln 3}$

$=3^a/\ln 3$

$\therefore \int 3^{\sqrt{2x+1}} = v.u-\int v\,du$ (00½)

$=\frac{a.3^a}{\ln 3}-\int\frac{3^a}{\ln 3}da$ —— (iii)

$\Rightarrow$ Let $M=3^a$.

$\ln M=a\ln 3$.

$\frac{dM}{M}=\ln 3\,da$.

$da=\frac{1}{\ln 3}\frac{dM}{M}$.

$\int 3^{\sqrt{2x+1}}dx=\frac{aM}{\ln 3}-\frac{\int}{\ln 3}.M\,da$ 0

[b] From (iii) $\int 3^{\sqrt{2x+1}}dx = \frac{a\,3^a}{\ln 3} - \int \frac{3^a}{\ln 3}\,da$

and $M = 3^a$.

$= \frac{aM}{\ln 3} - \int \frac{M}{\ln 3}\cdot da$ (01/2)

$= \frac{aM}{\ln 3} - \frac{M}{\ln 3}\int da$. also $da = \frac{1}{M\ln 3}\,dM$

By substituting

$= \frac{aM}{\ln 3} - \frac{M}{\ln 3}\cdot\frac{1}{M\ln 3}\int dM$

$= \frac{aM}{\ln 3} - \frac{1}{(\ln 3)^2}M + C$

$= \frac{a\cdot 3^a}{\ln 3} - \frac{1}{(\ln 3)^2}\cdot 3^a + C$ (01/2)

$= 3^a\left[\frac{a}{\ln 3} - \frac{1}{(\ln 3)^2}\right] + C$

$= 3^{\sqrt{2x+1}}\left[\frac{\sqrt{2x+1}}{\ln 3} - \frac{1}{(\ln 3)^2}\right] + C$

$= \frac{3^{\sqrt{2x+1}}}{\ln 3}\left(\sqrt{2x+1} - \frac{1}{\ln 3}\right) + C$ (01/2)

$\therefore \int 3^{\sqrt{2x+1}}dx = \frac{3^{\sqrt{2x+1}}}{\ln 3}\left(\sqrt{2x+1} - \frac{1}{\ln 3}\right) + C$

9(c) (i) $y = x(3-x)$ by $y = x$.

Solving

$y = x(3-x)$ —— (i)

$y \Rightarrow y = x$ —— (ii)

$x = x(3-x)$

$x^2 - 2x = 0$

10 $x(x-2) = 0$

$x = 0$ or 2

let $y_1 = 3x - x^2$. (00½)

10 $y_2 = x$. (00½)

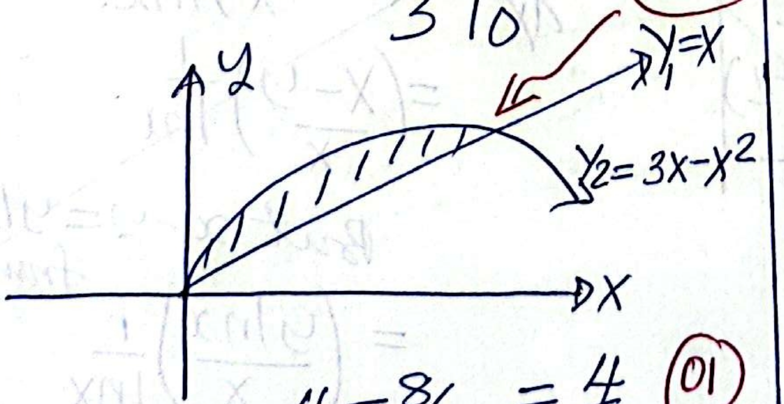
Area Enclosed

$$A = \int_0^2 (y_1 - y_1)\,dx.$$

$$= \int_0^2 (3x - x^2 - x)\,dx$$

$$= \int_0^2 (2x - x^2)\,dx$$

$$= x^2 - \frac{x^3}{3}\Big|_0^2$$ (00½)



$$= 4 - \tfrac{8}{3} = \frac{4}{3}$$ (01)

Area Enclosed $= 1\tfrac{1}{3}$ Sq. units

(ii) $\displaystyle\int_{-2}^{-1} \frac{dx}{2x^2+6x+5}$

$2x^2+6x+5 = 2(x^2+3x)+5$

$= 2(x^2+3x+\tfrac{9}{4}) + 5 - \tfrac{9}{2}$

$= 2(x+\tfrac{3}{2})^2 + \tfrac{1}{2}$

$$\int_{-2}^{-1} \frac{dx}{2x^2+6x+5} = \int_{-2}^{-1} \left(\frac{dx}{2(x+\frac{3}{2})^2+\frac{1}{2}}\right).$$

let $2(x+\tfrac{3}{2})^2 = \tfrac{1}{2}\tan^2\theta$.

From. $\left[\sqrt{2}(x+\tfrac{3}{2})\right] = \tan\theta$.

$(x+\tfrac{3}{2}) = \tfrac{1}{2}\tan\theta$. (01)

$\frac{dx}{d\theta} = \frac{1}{2}\sec^2\theta$

$dx = \frac{1}{2}\sec^2\theta\,d\theta$

$$\int_{-2}^{-1} \frac{\frac{1}{2}\sec^2\theta\,d\theta}{\frac{1}{2}\tan^2\theta + \frac{1}{2}} = \int \frac{dx}{2(x+\frac{3}{2})^2+\frac{1}{2}}$$

$$\int \frac{\sec^2\theta\,d\theta}{\tan^2\theta+1} = (\quad)$$

$$\int \frac{\sec^2\theta\,d\theta}{\sec^2\theta}$$ (01)

$\Rightarrow \theta\Big]_{-2}^{-1}$

$= \tan^{-1}(2x+3)\Big]_{-2}^{-1}$

$= \tan^{-1}1 - \tan^{-1}(-1)$

$= \frac{\pi}{4} - (\frac{\pi}{4})$ (00½)

$$\int_{-2}^{-1} \frac{1}{2x^2+6x+5}\,dx = \frac{\pi}{2}$$

10(a)(a) Given $Z = \frac{x^2}{x-y+1}$ → By Quotient Rule.

$$\frac{\partial z}{\partial x} = \frac{(x-y+1)2x - x^2}{(x-y+1)^2}$$ (01)

$$= \frac{2x^2 - 2xy + 2x - x^2}{(x-y+1)^2}$$

$$= \frac{x^2 - 2x(y-1)}{(x-y+1)^2}$$ (01)

$$\frac{\partial z}{\partial x} = \frac{x^2}{(x-y+1)^2} - \frac{2x(y-1)}{(x-y+1)^2}$$ (01)

(10)

(ii) $x^y = e^{x-y}$ Apply lin ln both sides

$y \ln x = x - y$ —— (1)

$y \ln x + y = x$ ⇒ $y \cdot \frac{1}{x} + \ln x \frac{dy}{dx} = 1$.

$y(\ln x + 1) = x$

$y = \frac{x}{\ln(x+1)}$ ⇒ $\frac{dy}{dx} = (1 - \frac{y}{x})\frac{1}{\ln x}$.

$\frac{dy}{dx} = \ln(x+1) -$ $= \left(\frac{x-y}{x}\right)\frac{1}{\ln x}$.

But $x - y = y \ln x$ from (1)

$= \left(\frac{y \ln x}{x}\right)\frac{1}{\ln x}$

(10) (10) (10) (30)

10(b) Given. $\sin xy + \frac{x}{y} = x^3 - y$.

Taking derivatives w.r.t. x.

$y\cos xy + \frac{d}{dx}(xy^{-1}) = 3x^2 - \frac{dy}{dx}$. --- (01)

$y\cos xy + \frac{1}{y}\frac{dx}{dx} + x\frac{d}{dx}(y^{-1}) = 3x^2 - \frac{dy}{dx}$.

$y\cos xy + \frac{1}{y} - xy^{-2}\frac{dy}{dx} = 3x^2 - \frac{dy}{dx}$.

$y\cos xy + \frac{1}{y} - 3x^2 = \frac{x}{y^2}\frac{dy}{dx} - \frac{dy}{dx}$.

$y\cos xy + \frac{1}{y} - 3x^2 = \frac{dy}{dx}\left(\frac{x}{y^2} - 1\right)$. --- (01)

$$\frac{dy}{dx} = \frac{y\cos xy + \frac{1}{y} - 3x^2}{\frac{x}{y^2} - 1}.$$ (01)

OR

$$\frac{dy}{dx} = \frac{y^3\cos xy - y - 3x^2y^2}{x - y^2}$$ By common denominator

10(c) $\frac{dv}{dt} = 900 cm^3/s$

$V = \frac{4}{3}\pi r^3$. Volume of Sphere.

But

$\frac{dv}{dt} = \frac{dv}{dr} \cdot \frac{dr}{dt}$ ——— (1) (00½)

If $V = \frac{4}{3}\pi r^3$.

$\frac{dv}{dr} = 4\pi r^2$ (00½)

$\frac{dv}{dt} = 4\pi r^2 \cdot \frac{dr}{dt}$ (01)

$900 = 4\pi r^2 \cdot dr/dt$.

$\frac{dr}{dt} = 900/4\pi r^2$ when $r = 15$ cm. (01)

$= \frac{900}{4\pi(15)^2}$

$= \frac{900\ cm}{900\pi\ s}$ (01)

∴ The rate of radius increase $= \frac{1}{\pi}$ cm/s.

CamScanner