PROPOSED MARKING GUIDE
ADVANCED MATHS 2

1(a)(i) MATHEMATICS : Has 11 letters $N = 11$
MATHS : Has 5 letters $r = 5$

No. of arrangement of 5 letters differently from 11 letters : $^{N}P_{r}$ ways.

Arrangement:

$$^{11}P_5 = \frac{11!}{(11-5)!}$$

$$= \frac{11!}{6!} = \frac{11 \times 10 \times 9 \times 8 \times 7 \times 6!}{6!}$$

There are $= 55,440$ ways. 02

(ii) $P(A) = 2/3$, $P(B) = 1/4$, $P(A/B) = 2/5$

$$P(A/B) = \frac{P(A \cap B)}{P B} \text{ —— } (1)$$

But $P(B) = P(B \cap A') + P(B \cap A)$

$P(B \cap A) = P(B) - P(B \cap A')$ —— (ii) 01

Substitute (ii) into (i)

$$P(A/B) = \frac{P(B) - P(B \cap A')}{P(B)}$$

$$\therefore P(B \cap A') = P(B) - P(A/B) \cdot P(B)$$

$$= 1/4 - (2/5)(1/4)$$ 01

01

1 $P(B\cap A') = \frac{1}{4}(1-\frac{2}{5}) = \frac{1}{4}(\frac{3}{5})$

$\therefore P(B\cap A') = 3/20$. 01

(b) (i) from $\int_a^b f(x)\,dx = 1$, $1 \le x \le 3$.

$\int_1^3 (cx^2+e^x)\,dx = 1$.

$\left[c\frac{x^3}{3}+e^x\right]_1^3 = 1$.

$(9c+e^3)-(\frac{k}{3}+e)=1$ 01

$c(9-\frac{1}{3})+e^3-e=1$.

$c = \frac{1-[e^3-e]}{26/3} = \frac{3(1+e-e^3)}{26}$.

$c = -1.8885$ 01

(ii) $P(0 \le x \le 0.5) = \int_0^{0.5}(-1.8885x^2+e^x)\,dx = \int_0^{0.5}(-1.8885x^2+1)\,dx$

$= \left.-1.8885\frac{x^3}{3} + \cancel{e^x}\right]_0^{0.5}$ for $e^x = 1$. 01

$= \left[-1.8885\frac{x^3}{3}+x\right]_0^{0.5}$ 02

$\therefore P(0 \le x \le 5) = 0.421$ 01

02

$n = 400, P = 0.1$

(C) (i) Mean (μ) $= np$

$= (400)(0.1)$

Variance $\delta^2 = npq$ but $q = 1-p$

$= np(1-p)$

$= 40(0.9)$

$= 36.$

Variance $= 36$ ------------ 01

$\therefore$ Mean $= 40$ ------------ 01

(ii) Given Averager rate $= \frac{600}{60\,min} = 10$ car/min

Mean/Average (λ) $= 10$

Let x be the r. variable repsent a car

$$P(x) = \frac{e^{-\lambda}\lambda^{x}}{x!}$$ (when $x = 1$ car, $\lambda = 10$

$$P(1) = \frac{e^{-10} \cdot 10^{1}}{1!}$$ 01

$$= \frac{10}{e^{10}}$$

$= 0.00045399.$

01

$\therefore$ Probability of passing a car $= 0.00045$

03

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2(b) $(P \leftrightarrow Q) \rightarrow (P \rightarrow Q)$ given

$[(P \rightarrow Q) \wedge (Q \rightarrow P)] \rightarrow (P \rightarrow Q)$ By definition

$\sim[(\sim P \vee Q) \wedge (\sim Q \vee P)] \vee (\sim P \vee Q)$ By def.

$[\sim(\sim P \vee Q) \vee \sim(\sim Q \vee P)] \vee (\sim P \vee Q)$ DeMorgans.

$\sim(\sim P \vee Q) \vee (\sim P \vee Q) \vee \sim(\sim Q \vee P)$ Commutative.

02

Let $\sim P \vee Q = A$

$\sim A \vee A \vee \sim(\sim Q \vee P)$

$T \vee (Q \wedge \sim P)$ complement, Demorgans law

$T \vee (Q \wedge \sim Q)$ complement law.

T identity law.

02

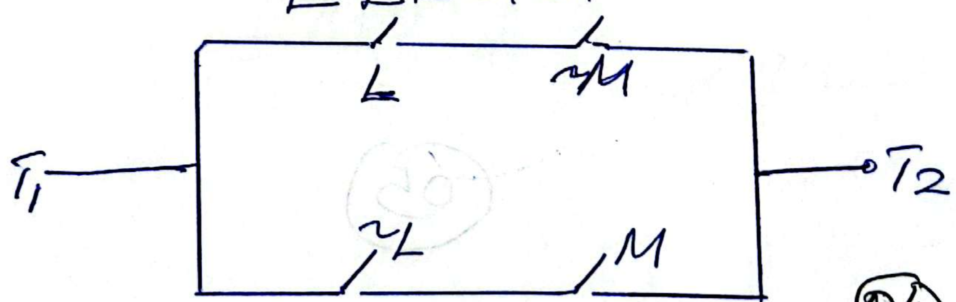
C FROM TRUTH TABLE

L	M	Z
T	T	F
T	F	T
F	T	T
F	F	F

ON T CIRCUIT.

$\Rightarrow (L \wedge \sim M) \vee (\sim L \wedge M)$

ELECTRI NETWORK.



05

04

2(a) P = Mr. Busara derives mathematical Eqns
Q = he becomes competent in Mathematics

conditional statement:

s: $p \to q$

convers: $q \to p$.

(i) $q \to p \equiv$ If Mr. Busara becomes competent in Mathematics then he derives Mathematical Equation 01 01

(ii) Contrapositive: $\sim q \to \sim p$.

$\therefore \sim q \to \sim p \equiv$ If Mr. Busarar won't be competent in Mathematics then he does not derive Mathematical equation 01 01

(iii) Inverse: $\sim p \to \sim q$

$\sim p \to \sim q \equiv$ Mr. Busara does not derive Mathematic Equation then he can not become competent in Mathematics 01 01

05

3(a) Given $\underset{\sim}{a} = i + 3j$

(i) Let $\underset{\sim}{b} = \underset{\sim}{v}$ whose magnitude

$|V| = 12$.

For parallel vector

$\hat{b} = \hat{a}$

$\frac{V}{|V|} = \frac{a}{|a|} \Longleftrightarrow \frac{V}{12} = \frac{i+3j}{\sqrt{1+9}}$ ------ (01)

$V = \frac{12(i+3j)}{\sqrt{10}}$

$= \frac{12\sqrt{10}}{10}(i+3j)$. Rationalized

$\therefore$ Vector $(\underset{\sim}{V}) = \frac{6\sqrt{10}}{5}(\underset{\sim}{i} + 3\underset{\sim}{j})$ ---- (01)

(ii) Area of parallelogram $= |\underset{\sim}{a} \times \underset{\sim}{b}|$, $a = 2i + k$, $b = i - 3j + k$.

$$A = \left\| \begin{matrix} i & j & k \\ 2 & 0 & 1 \\ 1 & -3 & 1 \end{matrix} \right\|$$

$= |i(3) - j(2-1) + k(-6)|$ (02)

$= |3i - j - 6k|$

$= \sqrt{9 + 1 + 36}$

$= \sqrt{46}$ square units (01)

(06)

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3(b) Given $\underline{a} = \cancel{2i+k}$ $2i+3j+4k$.

$\underline{b} = \cancel{4i+3j+k}$ $4i+5j+4k$

$f = 2i-j+10k$

where f – direction force

$f_a = (4i-3j+2k)\,N$ (01)

Work done $= F \cdot d$

But

$F = |f_a|\hat{f}$

$= |f_a| \cdot \frac{f}{|f|}$ (01)

$= |4i-3j+2k| \left(\frac{2i-j+10k}{\sqrt{4+1+100}}\right)$ (01)

$= \frac{\sqrt{29}}{\sqrt{105}}(2\underline{i}-\underline{j}+10\underline{k})$.

Work done $= \frac{\sqrt{29}}{\sqrt{105}}\begin{pmatrix}2\\-1\\10\end{pmatrix} \cdot (d)$ (01)

But $d = \begin{pmatrix}4\\5\\4\end{pmatrix} - \begin{pmatrix}2\\3\\4\end{pmatrix}$

$= \begin{pmatrix}2\\2\\0\end{pmatrix}$

$= \frac{\sqrt{29}}{\sqrt{105}}\begin{pmatrix}2\\-1\\10\end{pmatrix} \cdot \begin{pmatrix}2\\2\\0\end{pmatrix}$

$= \frac{\sqrt{29}}{\sqrt{105}}(4-2+0)$ (01)

W.D. $= \frac{2\sqrt{29}}{\sqrt{105}}$ or 1.051 Joule

(07)

3(c) (i) $r(t) = (2t^2-9)i + (3t^4-\sqrt{24}t)j - 9k$

$\text{speed} = \frac{d}{dt}[r(t)] = \frac{d}{dt}(2t^2-9)i + (3t^4-\sqrt{24}t)j - 9k$. 01

$= 4ti + (12t^3-\sqrt{24})j + 0k$ — (1)

$r'(t) = 4ti + (12t^3-\sqrt{24})j$

$\frac{d}{dt}[r'(t)] = \frac{d}{dt}(4ti + (12t^3-\sqrt{24})j$ 01

$r''(t) = 4i + 36t^2j$

when $t=2$.

$r''(2) = 4i + 36(2^2)j$

$r''(2) = 4i + 144j$ 01

(ii) Speed $\Rightarrow r'(t) = 4ti + (12t^3-\sqrt{24})j$ 01

But $t = x$

$r'(x) = 4xi + (12x^3-\sqrt{24})j$ 01

08

4(a) $Z = \frac{4+3i}{2-i}$

$= \left(\frac{4+3i}{2-i}\right)\left(\frac{2+i}{2+i}\right) = \frac{8+4i+6i-3}{4+1} = \frac{5+10i}{5}$

$= \frac{5(1+2i}{5}$

$Z = 1+2i$ ---------- 01

$|Z| = \sqrt{1+2^2}$ ---------- 01

$|Z| = \sqrt{5}$

$Arg z = \tan^{-1}\left(\frac{2}{1}\right)$ ---------- 01

$\therefore Arg z = 63.43°$

(ii) Multiplicative inverse of $Z = \frac{1}{z}$.

$\frac{1}{z} = \frac{1}{\frac{4+3i}{2-i}}$

$= \frac{2-i}{4+3i}$ ---------- 01

$= \left(\frac{2-i}{4+3i}\right)\left(\frac{4-3i}{4-3i}\right)$

$= \frac{8-6i-4i+3}{16+9} = \frac{11-10i}{25}$

$\therefore$ Multiplicative inverse of $Z = \frac{11}{25} - \frac{2}{5}i$ 01

09

(b

1(b) Let $a = 1+i \Rightarrow |a| = \sqrt{2}$, $\text{Arg}\, a = \tan^{-1} 1 = \pi/4$.

$b = \sqrt{3}+i \Rightarrow |b| = 2$, $\text{Arg}\, b = \tan^{-1}\frac{1}{\sqrt{3}} = \pi/6$. (01)

$$Z = \frac{(1+i)^3}{(\sqrt{3}+i)^2} = \frac{\left(\sqrt{2}[\cos\pi/4 + i\sin\pi/4]\right)^3}{\left(2[\cos\pi/6 + i\sin\pi/6]\right)^2}$$

$$= \frac{\sqrt{2}[\cos 3\pi/4 + i\sin 3\pi/4]}{2[\cos 2\pi/6 + i\sin 2\pi/6]}$$ By Demorg.

$$= \frac{\sqrt{2}(\cos 3\pi/4 + i\sin 3\pi/4)}{2(\cos\pi/3 + i\sin\pi/3)}$$

Rationalize denominator

$$= \frac{\sqrt{2}(\cos 3\pi/4 + i\sin 3\pi/4)(\cos\pi/3 - i\sin\pi/3)}{2(\cos\pi/3 + i\sin\pi/3)(\cos\pi/3 - i\sin\pi/3)}$$

$$= \frac{\sqrt{2}\left(\cos\frac{3\pi}{4}\cos\frac{\pi}{3} - i\cos\frac{3\pi}{4}\sin\frac{\pi}{3} - i\sin\frac{3\pi}{4}\cos\frac{\pi}{3} + \sin\frac{3\pi}{4}\sin\frac{\pi}{3}\right)}{2\left(\cos^2\frac{\pi}{3} + \sin^2\frac{\pi}{3}\right)}$$

$$= \frac{\sqrt{2}}{2}\left(\cos\frac{3\pi}{4}\cos\frac{\pi}{3} + \sin\frac{3\pi}{4}\sin\frac{\pi}{3}\right) - i\left(\sin\frac{3\pi}{4}\cos\frac{\pi}{3} + \sin\frac{\pi}{3}\cos\frac{3\pi}{4}\right)$$

$$= \frac{\sqrt{2}}{2}\left(\cos\left(\frac{3\pi}{4} - \frac{\pi}{3}\right) - i\sin\left(\frac{3\pi}{4} + \frac{\pi}{3}\right)\right)$$ 1

$$= \frac{\sqrt{2}}{2}\left(\cos\frac{5\pi}{12} - i\sin\frac{5\pi}{12}\right)$$

better 2nd ALTERNATIVE

Use 2

10

ALTERNATIVES

From

$$Z = \frac{\sqrt{2}\left(\cos \pi/4 + i \sin \pi/4\right)^3}{2\left(\cos \pi/6 + i \sin \pi/6\right)^2}$$

10

$$= \frac{\sqrt{2}}{2}\left(\cos \pi/4 + i \sin \pi/4\right)^3\left(\cos \pi/6 + i \sin \pi/6\right)^{-2}$$

60

$$= \frac{\sqrt{2}}{2}\left(\cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4}\right)\left(\cos\left(-\frac{\pi}{3}\right) + i \sin\left(-\frac{\pi}{3}\right)\right)$$

01

$$= \frac{\sqrt{2}}{2}\left[\cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4}\right]\left[\cos \pi/3 - i \sin \pi/3\right]$$

$$= \frac{\sqrt{2}}{2}\left(\cos \frac{3\pi}{4} \cos \frac{\pi}{3} + i \sin \frac{3\pi}{4} \cos \frac{\pi}{3} - i \cos \frac{3\pi}{4} \sin \frac{\pi}{3} + \sin \frac{3\pi}{4} \sin \frac{\pi}{3}\right)$$

01

10

$$= \frac{\sqrt{2}}{2}\left(\cos \frac{3\pi}{4} \cos \frac{\pi}{3} + \sin \frac{3\pi}{4} \sin \pi/3\right) + i\left(\sin \frac{3\pi}{4} \cos \frac{\pi}{3} - \cos \frac{3\pi}{4} \sin \frac{\pi}{3}\right)$$

$$= \frac{\sqrt{2}}{2}\left(\cos\left(\frac{3\pi}{4} - \frac{\pi}{3}\right)\right) + i\left(\sin\left(\frac{3\pi}{4} - \frac{\pi}{3}\right)\right)$$

10

$$= \frac{\sqrt{2}}{2}\left(\cos \frac{5\pi}{12} + i \sin \frac{5\pi}{12}\right)$$

01

Also In Eulers formula

$$Z = \frac{\sqrt{2}}{2} e^{\frac{5\pi}{12} i}$$

01

11

4(c) To prove $\sin^5\theta = \frac{1}{16}[\sin 5\theta - 5\sin 3\theta + 10\sin\theta]$

From

$$2i\sin\theta = z - \frac{1}{z}$$

$$\sin\theta = \frac{1}{2i}\left(z - \frac{1}{z}\right) \quad 01$$

$$(2i\sin\theta)^5 = \left(z - \frac{1}{z}\right)^5$$

$$32i\sin^5\theta = z^5 - 5z^4\cdot\frac{1}{z} + 10z^3\cdot\frac{1}{z^2} - 10z^2\cdot\frac{1}{z^3} + \frac{5z}{z^4} - \frac{1}{z^5}$$

$$= \left(z^5 - \frac{1}{z^5}\right) - 5\left(z^3 - \frac{1}{z^3}\right) + 10\left(z - \frac{1}{z}\right) \quad 02$$

$$= (2i\sin 5\theta) - 5(2i\sin 3\theta) + 10(2i\sin\theta)$$

$$= 2i(\sin 5\theta - 5\sin 3\theta + 10\sin\theta) \quad 01$$

$$\sin^5\theta = \frac{2i}{32i}(\sin 5\theta - 5\sin 3\theta + 10\sin\theta)$$

$$\therefore \sin^5\theta = \frac{1}{16}[\sin 5\theta - 5\sin 3\theta + 10\sin\theta] \quad 01$$

Required.

12

SECTION B

5 (a) Consider $f(\theta) = 3\cos\theta - 4\sin\theta + 6$

Taking

$3\cos\theta - 4\sin\theta = R\sin(\theta - \alpha)$

$3\cos\theta - 4\sin\theta = R\sin\alpha.\cos\theta - R\cos\alpha\sin\theta$

Compare

01

$3\cos\theta = R\sin\alpha\cos\theta.$

$3 = R\sin\alpha.$ —— (I)

$4 = R\cos\alpha$ —— (II)

Dividing (I) with (II)

$\frac{\sin\alpha}{\cos\alpha} = 3/4 \Rightarrow \alpha = \tan^{-1} 3/4.$

$\alpha = 36.87^{\circ}$

01

At maximum

$\sin(\theta - \alpha) = 1.$

But $R^2(\sin^2\alpha + \cos^2\alpha) = 3^2 + 4^2$

$R = \sqrt{25} = 5$

$\therefore 3\cos\theta - 4\sin\theta = 5\sin(\theta - 36.87^{\circ})$

$= 5(1).$

$3\cos\theta - 4\sin\theta + 6 = 5 + 6$

The maximum value of $f(\theta)$ or

$3\cos\theta - 4\sin\theta + 6 = \underline{11}$

01

FOR MINIMUM

$\sin(\theta - 36.87^{\circ}) = -1$

$5\sin(\theta - 36.87) = -5$

5(a) Adding 6 bothsides

$5\cos(\theta-36.87)+6=-5+6.$

The minimum value $= 1$.

The corresponding angle:

From.

Max:

$\sin(\theta-36.87^\circ)=1.$

$5\sin(\theta-36.87)=5$

$5\sin(\theta-36.87)+6=11$

Solving for θ:

$\Rightarrow 5\sin(\theta-36.87^\circ)=11-6.$

$\sin(\theta-36.87)=1$

$\theta-36.87=\sin^{-1}1$

$\theta=90^\circ+36.87$ 01

$\theta=126.87^\circ.$

For minimum.

$\sin(\theta-36.87)=-1$

$\theta-36.87=-90^\circ$

$\theta=-53.13$ 01

$\therefore$ The maximum angle $\theta=126.87$ and minimum corr. angle $=-53.13$.

14

b(i). $\sin 20^\circ \sin 40^\circ \sin 80^\circ = \sin 20^\circ(\sin 40^\circ \sin 80^\circ)$

$$= \sin 20^\circ\left[\tfrac{-1}{2}(\cos 120 - \cos 40)\right]$$

from $2\sin 40 \sin 80 = -\cos(40+8) + \cos(80-40)$

$$= \sin 20^\circ\left[\tfrac{1}{2}\left(-\tfrac{1}{2} - \cos 40\right)\right] \quad 01$$

$$= \sin 20^\circ\left[\tfrac{1}{4} + \tfrac{1}{2}\cos 40\right]$$

$$= \tfrac{1}{4}\sin 20 + \tfrac{1}{2}\sin 20 \cos 40$$

But $\sin 20 \cos 40 = ?$

$$\sin 20 \cos 40^\circ = \tfrac{1}{2}(\sin(20+40) - \sin(40-20))$$

$$= \tfrac{1}{2}(\sin 60 - \sin 20^\circ)$$

$$= \tfrac{1}{4}\sin 20^\circ + \tfrac{1}{2}\left[\tfrac{1}{2}(\sin 60^\circ - \sin 20^\circ)\right] \quad 01$$

$$= \tfrac{1}{4}\sin 20^\circ + \tfrac{1}{2}\left[\tfrac{1}{2}\left(\tfrac{\sqrt{3}}{2} - \sin 20^\circ\right)\right].$$

$$= \cancel{\tfrac{1}{4}\sin 20^\circ} + \tfrac{\sqrt{3}}{8} - \cancel{\tfrac{1}{4}\sin 20^\circ}$$

$$= \frac{\sqrt{3}}{8} \text{ Proved.} \quad 01$$

15

5

b

(i) $\dfrac{\cos 4\theta + \cos 3\theta + \cos 2\theta}{\sin 4\theta + \sin 3\theta + \sin 2\theta} = \cot 3\theta$

LHS

$$\frac{(\cos 4\theta + \cos 2\theta) + \cos 3\theta}{(\sin 4\theta + \sin 2\theta) + \sin 3\theta}$$ 01

$$\frac{\left[2\cos\left(\frac{4\theta+2\theta}{2}\right)\cos\left(\frac{4\theta-2\theta}{2}\right)\right] + \cos 3\theta}{\left[2\sin\left(\frac{4\theta+2\theta}{2}\right)\cos\left(\frac{4\theta-2\theta}{2}\right)\right] + \sin 3\theta}$$ 01

$$\frac{\cos 3\theta \cos\theta + \cos 3\theta}{\sin 3\theta \cos\theta + \sin 3\theta}$$

$$\frac{\cos 3\theta(\cos\theta + 1)}{\sin 3\theta(\cos\theta + 1)}$$

$$= \frac{\cos 3\theta}{\sin 3\theta}$$ 01

$= \cot 3\theta$ Proved.

16

5 (c) $\sin\theta + \sqrt{3}\cos\theta = 1 \Rightarrow \sin\theta + \sqrt{3}\cos\theta = 1.$

Let $\tan\theta/2 = t$

$\sin\theta = \dfrac{2t}{1+t^2}$

$\cos\theta = \dfrac{1-t^2}{1+t^2}$

By subst.

$$\frac{2t}{1+t^2} + \sqrt{3}\left(\frac{1-t^2}{1+t^2}\right) = 1.$$ 01

$$2t + \sqrt{3}(1-t^2) = 1+t^2.$$

$$2t + \sqrt{3} - \sqrt{3}t^2 - t^2 - 1 = 0$$

$$-2t - \sqrt{3} + \sqrt{3}t^2 + t^2 + 1 = 0.$$

$$(\sqrt{3}+1)t^2 - 2t + 1 - \sqrt{3} = 0.$$

$$t = \frac{2 \pm \sqrt{4 - 4(\sqrt{3}+1)(1-\sqrt{3})}}{2(\sqrt{3}+1)} \Rightarrow$$

$$t = \frac{2 \pm 2\sqrt{1-(1-3)}}{2(\sqrt{3}+1)}$$ 01

$$= \frac{1 \pm \sqrt{1 - -2}}{\sqrt{3}+1}$$

$$= \frac{1 \pm \sqrt{3}}{\sqrt{3}+1}.$$

$$t = \frac{1+\sqrt{3}}{\sqrt{3}+1} \text{ or } \frac{1-\sqrt{3}}{\sqrt{3}+1}.$$ 01

But $\tan\frac{\theta}{2} = t$

5(c) $\tan\theta/2 = \frac{1+\sqrt{3}}{1+\sqrt{3}}$ or $\tan\theta/2 = \frac{1-\sqrt{3}}{1+\sqrt{3}}$

$\frac{\theta}{2} = \tan^{-1}1$ or $\theta = \tan^{-1}\left(\frac{1-\sqrt{3}}{1+\sqrt{3}}\right)$

$= \tan^{-1}\left[\left(\frac{1-\sqrt{3}}{1+\sqrt{3}}\right)\left(\frac{1-\sqrt{3}}{1-\sqrt{3}}\right)\right]$

$= \tan^{-1}\left(\frac{4-2\sqrt{3}}{-2}\right)$

$= \tan^{-1}\left(\frac{2-\sqrt{3}}{-1}\right)$

$= \tan^{-1}(\sqrt{3}-2)$ 01

$\frac{\theta}{2} = \pi/4$ or $\pi/12$

General formula:

$\frac{\theta}{2} = \pi n + \pi/4$ or $\pi n + \pi/12$

$\therefore \theta = \left(\frac{1}{2}\pi n + \pi/8\right)$ or $\frac{\pi n}{2} + \pi/24$ 01

18

5 (d) Given $\cos A + \cos B + \cos C - 1 = 4\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$

LHS.

from $A+B+C = 180$

$A+B = 180 - C$

$\frac{A+B}{2} = 90 - \frac{C}{2}$ ——— (1)

$\cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$ 01

Then $[\cos A + \cos B] + \cos C - 1$

$2\cos\frac{A+B}{2}\cos\left(\frac{A-B}{2}\right) + \cos C - 1$

But $\cos\left(\frac{A+B}{2}\right) = \cos\left(90 - \frac{C}{2}\right)$ (11)

$2\cos\left(90 - \frac{C}{2}\right)\cos\left(\frac{A-B}{2}\right) + \cos C - 1$

since $\cos\left(90° - \frac{C}{2}\right) = \sin\frac{C}{2}$

$\cos C = \cos^2\frac{C}{2} - \sin^2\frac{C}{2}$

$2\sin\frac{C}{2}\cos\left(\frac{A-B}{2}\right) + \cos^2\frac{C}{2} - \sin^2\frac{C}{2} - 1$ 01

and $\cos^2\frac{C}{2} + \sin^2\frac{C}{2} = 1$.

$2\sin\frac{C}{2}\cos\left(\frac{A-B}{2}\right) + \cos^2\frac{C}{2} - \sin^2\frac{C}{2} - \left(\cos^2\frac{C}{2} + \sin^2\frac{C}{2}\right)$.

$2\sin\frac{C}{2}\cos\left(\frac{A-B}{2}\right) - 2\sin^2\frac{C}{2}$

$2\sin\frac{C}{2}\left[\cos\left(\frac{A-B}{2}\right) - \sin\frac{C}{2}\right]$; $\frac{C}{2} = \frac{180}{2} - \frac{A+B}{2}$

$= \frac{90}{2} - \left(\frac{A+B}{2}\right)$

$2\sin\frac{C}{2}\left[\cos\left(\frac{A-B}{2}\right) - \sin\left[90 - \left(\frac{A+B}{2}\right)\right]\right]$. 01

$2\sin\frac{C}{2}\left[\cos\left(\frac{A-B}{2}\right) - \cos\left(\frac{A+B}{2}\right)\right] \rightarrow$ AS (11) above

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5(d) $\Rightarrow 2\sin\frac{C}{2}\left[-2\sin\left(\frac{(A-B)+A+B}{2}\right]\sin\left[\frac{A-B-(A+B)}{2}\right)\right]$

$= 2\sin\frac{C}{2}\left[-2\sin\frac{A}{2}\left(-\sin\frac{B}{2}\right)\right]$ 01

From $\sin\left(-\frac{B}{2}\right) = -\sin\frac{B}{2}$
odd functn

$= 4\sin\frac{C}{2}\sin\frac{A}{2}\sin\frac{B}{2}$

$= 4\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$ (Commutative prop.) 01

$\therefore \cos A + \cos B + \cos C - 1 = 4\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$ Proved.

20

6(a) Given $\dfrac{x^5-4x^3+2x+3}{x+2}$

$$
\begin{array}{r}
x^4-2x^3+2 \qquad\qquad\qquad \\
x+2 \overline{\big) x^5+0x^4-4x^3+0x^2+2x+3} \\
-\;\; x^5+2x^4 \qquad\qquad\qquad\qquad\qquad \\
\hline
-2x^4-4x^3 \qquad\qquad\qquad \\
-\;\;(-2x^4-4x^3) \qquad\qquad\qquad \\
\hline
2x+3 \\
-\;\; 2x+4 \\
\hline
-1
\end{array}
$$

02

$\therefore$ The Remainder $= 1$.

01

b(i) $\dfrac{3x^2-x-1}{(x-1)5}$ To partialize

Let $y = x-1 \Rightarrow x = y+1$

$$\frac{3x^2-x-1}{(x-1)5} = \frac{3(y+1)^2-(y+1)-1}{y5}$$ 01

$$= \frac{3(y^2+2y+1)-y-1-1}{y5}$$

$$= \frac{3y^2+6y+3-y-1-1}{y5}$$ 01

21

$$= \frac{3y^2 + 5y + 1}{y^5}$$

$$= \frac{3y^2}{y^5} + \frac{5y}{y^5} + \frac{1}{y^5} \quad \text{But } y = x - 1$$

$$= \frac{3}{y^3} + \frac{5}{y^4} + \frac{1}{y^5}$$

$$= \frac{3}{(x-1)^3} + \frac{5}{(x-1)^4} + \frac{1}{(x-1)^5} \qquad 01$$

$$\therefore \frac{3x^2 - x - 1}{(x-1)^5} = \frac{3}{(x-1)^3} + \frac{5}{(x-1)^4} + \frac{1}{(x-1)^5} \qquad 01$$

6(ii) Given $(x-1)^3 = 5$

$$\frac{3x^2 - x - 1}{(x-1)^5} = \frac{3}{5} + \frac{5}{(x-1)5} + \frac{1}{(x-1)^3(x-1)^2} \qquad 01$$

$$= \frac{3}{5} + \frac{1}{x-1} + \frac{1}{5(x-1)^2}$$

But $(x-1)^3 = 5$

$x - 1 = \sqrt[3]{5}$ or $5^{1/3}$. 01

$$= \frac{3}{5} + \frac{1}{\sqrt[3]{5}} + \frac{1}{5\sqrt[3]{5^2}}$$

01

$$\therefore \frac{3x^2 - x - 1}{(x-1)^5} = \frac{3}{5} + \frac{1}{\sqrt[3]{5}} + \frac{1}{\sqrt[3]{25}} \text{ OR } \frac{3}{5} + \frac{1}{5^{1/3}} + \frac{1}{25^{1/3}}$$

6(c) Given $54x^3 - 111x^2 + 74x - 16 = 0$. Let $\frac{\alpha}{r}, \alpha, \alpha r$

for G.P: $\frac{G_2}{G_1} = \frac{G_3}{G_2} = r$

Sum of roots $\Rightarrow S = \frac{\alpha}{r} + \alpha + \alpha r = \frac{\alpha}{r}(1 + r + r^2) = -b/a$.

$$\Rightarrow \frac{\alpha}{r}(1+r+r^2) = +111/54$$ (00½)

$$\therefore \frac{\alpha}{r}(1+r+r^2) = 37/18 \quad \text{—— (I)}$$

Product of roots $P = \frac{\alpha}{r} \cdot \alpha \, \alpha r = \alpha^3 = -d/a$

(00½)

$$\therefore \alpha^3 = 16/54 \Rightarrow \alpha = 2/3 \quad \text{—— (II)}$$

Sum of prod of roots $SP = \frac{\alpha^2}{r} + \alpha^2 r + \alpha^2 = \frac{284}{64} = c/a$. —— (III) (00½)

Solving (2) and (1) result

$$\frac{2}{3r}(1+r+r^2) = 37/18$$

$$12r^2 - 25r + 12 = 0.$$ (00½)

$$r = 4/3 \text{ or } 3/4$$

case I: For $r = 4/3$, 1st root $= \frac{\alpha}{r} = \left(\frac{2}{3}\right)\left(\frac{3}{4}\right) = 1/2$

2nd root $\Rightarrow \alpha = 2/3$

3rd root $= \alpha r = \left(\frac{2}{3}\right)\left(\frac{4}{3}\right) = 8/9$.

Let P = Sum of squares of roots.

$$P = \left(\frac{1}{2}\right)^2 + \left(\frac{2}{3}\right)^2 + \left(\frac{8}{9}\right)^2$$

$$= \frac{1}{4} + 4/9 + \frac{64}{81}$$

$$P = \frac{1}{4} + \frac{4}{9} + \frac{64}{81}$$

$$= \frac{81+144+256}{324}$$

$$= \frac{481}{324}.$$

07

S = Product of squares of roots.

$$= \left(\frac{1}{4}\right)\left(\frac{4}{9}\right)\left(\frac{64}{81}\right) = \frac{64}{729}.$$

01

Then: $\frac{S}{P} = \frac{64}{729} \Big/ \frac{481}{324}$

$$= \frac{64}{729} \times \frac{324}{481}$$

$$\frac{S}{P} = \frac{20,736}{350,649}.$$

01

Case II: Arrange roots in decending: $\frac{8}{9}, \frac{2}{3}, \frac{1}{2}$

$P = \left(\frac{8}{9}\right)^2 + \left(\frac{2}{3}\right)^2 + \left(\frac{1}{2}\right)^2 = \frac{481}{324}.$

$S = \left(\frac{8}{9}\right)^2 \times \left(\frac{2}{3}\right)^2 \times \left(\frac{1}{2}\right)^2 = \frac{64}{729}$

Which gives the same value.

(Candidate may use either of them).

6(a) $x^3+px^2+qx+30=0$, Ratios $2:3:5$

Let roots are α, β, γ

But $\frac{\alpha}{k}=2, \frac{\beta}{k}=3, \frac{\gamma}{k}=5$

$\alpha=2k, \beta=3k, \gamma=5k.$ (01)

$$S = 2k+3k+5k=-p.$$

$$k(2+3+5)=-p$$

$$10k=-p \text{ ——— (i)}$$ (01)

$$SP=(2k\cdot 3k)+(2k)(5k)+(3k)(5k)=q.$$

$$6k^2+10k^2+15k^2=q$$

$$31k^2=q \text{ ——— (ii)}$$ (01)

$$P= (2k)(3k)(5k)=-30$$

$$30k^3=-30.$$

$$k=-1$$

Then using (i)

$$p=10$$

$$q=31(-1)^2.$$

$$q=31$$ (02)

The value of $p=10$ and $q=31$

25

7(a) 7(b) $x\frac{dy}{dx} = y[\ln y - \ln x + 1]$, Dividing by x.

$$\frac{dy}{dx} = \frac{y}{x}[\ln y - \ln x + 1]$$

$$= \frac{y}{x}\left[\ln\left(\frac{y}{x}\right) + 1\right] \text{ — ⓐ is Homog. DE.} \quad 01$$

Let $y = ux \Rightarrow y/x = u$ — ①

$$\frac{dy}{dx} = u + x\frac{du}{dx} \text{ — ⑪}$$

Substituting ① and ⑪ in ⓐ

$$u + x\frac{du}{dx} = u[\ln u + 1] \quad 01$$

$$u + x\frac{du}{dx} = u\ln u + u.$$

$$x\frac{du}{dx} = u\ln u.$$

Separating, Integrating

$$\int\frac{du}{u\ln u} = \int\frac{dx}{x}. \quad 01$$

$\ln(\ln u) = \ln x + C$ let $C = \ln A$ – Constant

$$\ln(\ln u) = \ln x + \ln A$$

$$\ln(\ln u) = \ln(Ax)$$

$$\ln u = Ax \quad 01$$

26

7a (i) $y = Ae^{bx+1}$

$= Ae \cdot e^{bx}$. But $Aee^{bx} = y$.

$\frac{dy}{dx} = Aebe^{bx}$

$\frac{dy}{dx} = b(Aee^{bx}) \Leftrightarrow \frac{dy}{dx} = by$. (01)

$\frac{dy}{dx}$

$\Rightarrow \frac{1}{y}\frac{dy}{dx} = b$, Taking derivative both sides

$\frac{d}{dx}\left(\frac{1}{y}\frac{dy}{dx}\right) = \frac{d}{dx}(b)$ (02)

$\Rightarrow \frac{1}{y}\frac{d^2y}{dx} - \frac{1}{y^2}\frac{dy}{dx}\cdot\frac{dy}{dx} = 0$

Factorizing $1/y$

$\therefore \frac{d^2y}{dx^2} - \frac{1}{y}\left(\frac{dy}{dx}\right)^2 = 0$ OR $\frac{d^2y}{dx^2} = \frac{1}{y}\left(\frac{dy}{dx}\right)^2$

(ii) $y = ax + cy$

$y - cy = ax \Leftrightarrow y(1-c) = ax$.

$y = \frac{a}{1-c}x$ —(i) Taking derivatives (01)

$\frac{dy}{dx} = \frac{a}{1-c}$ —(ii) From (i) $\frac{y}{x} = \frac{a}{1-c}$

substituting

$\therefore \frac{dy}{dx} = \frac{y}{x}$ or $\frac{dy}{dx} - \frac{y}{x} = 0$. (01)

27

Continue

7(b) $\ln u = Ax$, But $y/x = u$.

$\ln\left(\frac{y}{x}\right) = Ax$

$y/x = e^{Ax}$

$y = xe^{Ax}$ where A is Constant.

01

c. $\frac{d^2y}{dx^2} + \frac{dy}{dx} - 8y = 0$ —— (a)

Let ~~$y = Me^x$~~ $y = e^{Mx}$ —— (i)

$\frac{dy}{dx} = Me^{Mx}$ —— (ii)

$\frac{d^2y}{dx^2} = M^2e^{Mx}$ —— (iii) Substituting i, ii, iii in (a),

01

$M^2e^{Mx} + 2Me^{Mx} - 8e^{Mx} = 0$

$M^2 + 2M - 8 = 0$

Roots: $x_1 = 2$, $x_2 = -4$ for $x = M$

If $M_1 = 2$ and $M_2 = -4$

G.Soln $y = Ae^{M_1x} + Be^{M_2x}$.

$y = Ae^{2x} + Be^{-4x}$

01

But $y(0) = 1$ and $y'(0) = 3$.

7(c) $y = Ae^{2x} + Be^{-4x}$

$y(0) = 1$

$y(0) = A + B$

$1 = A + B$ —— (1)

$\frac{dy}{dx} = y'(x) = 2Ae^{2x} - 4Be^{-4x}$ (01)

$y'(0) = 2A - 4B$

$3 = 2A - 4B$ —— (ii)

Solving (1) and (ii) simultenously.

$A = 7/6$ - - - - - - (01)

$B = -1/6$.

Thus from $y = Ae^{2x} + Be^{-4x}$.

By Substitution

$y = \frac{7}{6}e^{2x} - \frac{1}{6}e^{-4x}$. (01)

OR

$y = \frac{1}{6}(7e^{2x} - e^{-4x})$

7(d). Rate of cooling $\propto$ Excess Temp over Surrounding.

$$\frac{d\theta}{dt} \propto (\theta - \theta_R)$$

$$\frac{d\theta}{dt} = -K(\theta - \theta_R)$$

$$\int_{\theta_0} \frac{d\theta}{\theta - \theta_R} = \int_0^t -K\,dt$$

$$\int_{200}^{T} \left(\frac{d\theta}{\theta - 10}\right) = \int_0^t -K\,dt$$

$$\left[\ln(\theta - 10)\right]_{200}^{T} = -K\left[t\right]_0^t$$

02

$$\ln(T - 10) - \ln(200 - 10) = -Kt$$

$$\ln\left(\frac{T-10}{190}\right) = -Kt$$

$$\log_e\left(\frac{T-10}{190}\right) = -Kt$$

$$\frac{T-10}{190} = e^{-Kt}$$

02

$$T = 10 + 190e^{-Kt} \quad \text{where } K = \frac{1}{40}\ln\left(\frac{19}{9}\right)$$

$\therefore$ Hence proved: $T = 10 + 190e^{-Kt}$ 01

30

8(a) For symmetric about y-axis and vertex at origin.

$X^2 = 4ay$

But passes at $P(x,y) = (6,2)$.

$6^2 = 4a2 \Leftrightarrow \frac{36}{8} = a$. (01)

$a = \frac{9}{2}$

Then Eqn of Parabola $\Rightarrow X^2 = 4 \cdot \frac{9}{2} y$

$X^2 = 18y$ or $X^2 - 18y = 0$ (02)

~~$-a = y$~~

$y =$

(ii) Equation of directrix: $y = -a$.

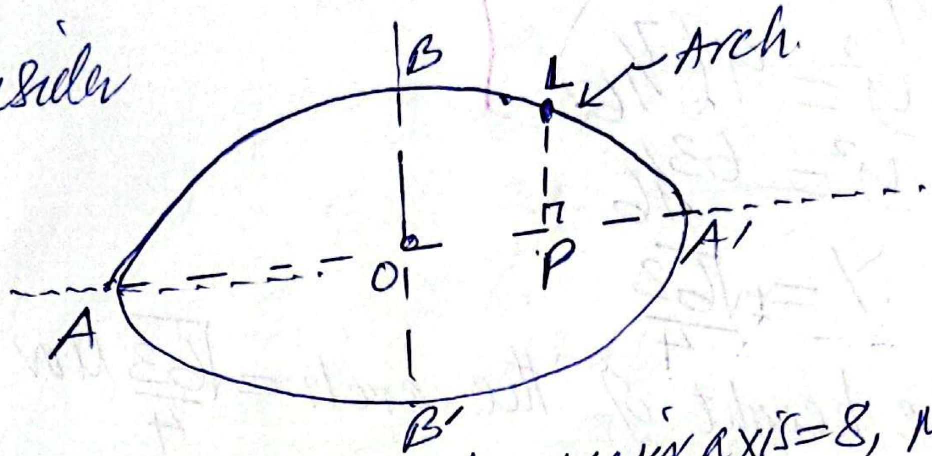
$y = \frac{-9}{2}$. (02)

Thus Eqn: $y + \frac{9}{2} = 0$ or $y = \frac{-9}{2}$

31

8(b). An Ellipse : Eqn $\frac{X^2}{a^2} + \frac{y^2}{b^2} = 1.$

consider



$00\frac{1}{2}$

Arch ABA' = Arch., Major axis = 8, minor = 3M.

AA' = 8M and OB = HEIGHT $00\frac{1}{2}$

2a = 8M OB = 3M. $00\frac{1}{2}$

a = 4M ✓ b = 3M. ✓

Then $\frac{X^2}{a^2} + \frac{y^2}{b^2} = \frac{X^2}{16} + \frac{y^2}{9} = 1.$

At point P(X, y) is 1 from end.

a = OA' = 4

PA' = 1

OP = OA' − PA'

= 4 − 1

∴ OP = 3 $00\frac{1}{2}$

Thus Coordinate at P(3,0) and L(3, y)

using Eqn. ✓ 01

$\frac{3^2}{16} + \frac{y^2}{9} = 1$

$$\frac{9}{16} + \frac{y^2}{9} = 1.$$

$$\frac{y^2}{9} = 1 - \frac{9}{16}.$$

$$y^2 = 9\left(\frac{7}{16}\right)$$

01

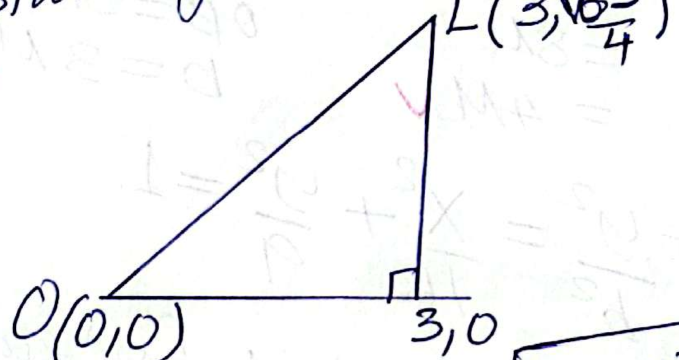
$$y^2 = \frac{63}{16}$$

01

$$y = \pm\frac{\sqrt{63}}{4}$$

The height of the arch $= \frac{\sqrt{63}}{4}$ M or 1.98M.

(ii) Distance from centre



$$\text{Distance } OL = \sqrt{(3-0)^2 + \left(\frac{\sqrt{63}}{4} - 0\right)^2}$$

$$= \sqrt{12.9375}$$

$$= \underline{\underline{3.5969 \text{ M.}}}$$

02

33

8(c) foci at $\pm(3\sqrt{6}, 0)$, Foci $= c$

Latus Rectum $= \frac{2b^2}{a}$

$10 = \frac{2b^2}{a}$

$10a = 2b^2$

$5a = b^2$ —— (I) 01

From $c = 3\sqrt{6}$

$c^2 = a^2 + b^2$

$(3\sqrt{6})^2 = a^2 + (5a)^2$ 01

$54 = a^2 + 25a^2$

$54 = 26a^2$

$a = \sqrt{\frac{54}{26}}$ or

$a^2 = \frac{54}{26}$ —— (II) 01

$b^2 = 5a$

$b^2 = 5\sqrt{\frac{54}{26}}$ —— (III) 01

Since foci lie on x-axis

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \Rightarrow \frac{x^2}{\frac{54}{26}} - \frac{y^2}{5\sqrt{\frac{54}{26}}} = 1.$

$= \frac{26x^2}{54} - \frac{\sqrt{26}\,y^2}{5\sqrt{54}} = 1.$ 01

34

8(d) In Polar form:

$$\frac{26x^2}{54} - \frac{\sqrt{26}}{5\sqrt{54}} y^2 = 1.$$

$x = r\cos\theta$ and $y = r\sin\theta$

$x^2 = r^2\cos^2\theta$ $\quad y^2 = r^2\sin^2\theta$ 02

Substituting

$$\frac{26r^2\cos^2\theta}{54} - \frac{\sqrt{26}}{5\sqrt{54}} r^2\sin^2\theta = 1.$$

$$r^2\left(\frac{26\cos^2\theta}{54} - \frac{\sqrt{26}}{5\sqrt{54}}\sin^2\theta\right) = 1$$ 03

OR

$$\frac{26}{54} r^2\cos^2\theta - \frac{\sqrt{26}}{5\sqrt{54}} r^2\sin^2\theta = 1.$$

8 (d)