MARKING GUIDE OF SERIES – 01

SET–01

01 (a) 49·1254

(b) 4

(c) $x_1 = 0.866$, $x_2 = -2.211$ and $x_3 = -3.655$

(d) 51·8802.

2 (a) Given $f(x) = x + \frac{1}{x}$

Req: To show that $[f(x)]^3 = f(x^3) + 3f(x)$

Consider L·H·S

$$[f(x)]^3 = \left(x + \frac{1}{x}\right)^3$$

$$= x^3 + 3x^2 \cdot \frac{1}{x} + 3x \cdot \frac{1}{x^2} + \frac{1}{x^3}$$

$$= x^3 + 3x + \frac{3}{x} + \frac{1}{x^3}$$

$$= x^3 + \frac{1}{x^3} + 3\left(x + \frac{1}{x}\right)$$

$$= f(x^3) + 3f(x)$$

Since L·H·S = R·H·S #

(b) Given:

$$f(x) = \begin{cases} x^2+1 & \text{for } x > 1 \\ |x| & \text{for } -2 < x \leq 1 \\ x+2 & \text{for } x \leq -2 \end{cases}$$

2 (b) Table of values

x	1	2	3	4
x^2+1	2	5	10	17

$|x|$ for $-2 < x \le 1$

x	-2	-1	0	1
$|x|$	2	1	0	1

$x+2$ for $x \le -2$

x	-2	-3	-4	-5
$x+2$	0	-1	-2	-3

~~Graph~~

(i) On graph paper.

(ii) Domain =

Range =

(iii) $f(-3) = -1$

$f(\frac{1}{2}) = \frac{1}{2}$

$f(2) = 5$

3 (a) Given: $x+y=8$ --- (i)

$xy=15$ --- (ii)

Req: x and y

Make y the subject from (i)

$y=8-x$ --- (iii)

Substitute eqn (iii) into (ii)

$x(8-x)=15$

$8x-x^2=15$

So, $-x^2+8x-15=0$

solve for x by calculator; $x=5$ or 3

But $y=8-x$

When $x=5$, $y=3$

$x=3$, $y=5$

$\therefore (x,y)=(5,3)$ or $(3,5)$

(b). Given: $G_1-G_2=4$

$G_2+G_3=\frac{8}{3}$

Req: G_1, G_2 and G_3

$G_1-G_1r=4 \Rightarrow G_1(1-r)=4$ --- (i)

also, $G_1r+G_1r^2=\frac{8}{3}$

$G_1(r+r^2)=\frac{8}{3}$ --- (ii)

Take eqn (i) $\div$ eqn (ii)

3 (b) $\frac{\cancel{G_1}(1-r)}{\cancel{G_1}(r+r^2)} = \frac{4}{8/3}$

$\frac{1-r}{r+r^2} = \frac{12}{8}$

$12r^2+12r = 8-8r$

so, $12r^2+20r-8=0$

Solve for r; $r = \frac{1}{3}$ or -2

For $r=\frac{1}{3}$	For $r=-2$
$G_1 = \frac{4}{1-r}$ $G_1 = 6$ also, $G_2 = G_1 \cdot r$ $G_2 = 6 \cdot \frac{1}{3} = 2$ also, $G_3 = G_2 \cdot r$ $= 2 \cdot (1/3) = 2/3$	$G_1 = \frac{4}{1-r}$ $G_1 = \frac{4}{3}$ $G_2 = G_1 r$ $= \frac{4}{3}(-2) = -\frac{8}{3}$ $G_3 = G_2 \cdot r = -\frac{8}{3}(-2) = \cancel{24}\ \frac{16}{3}$

$\therefore$ The three terms are 6, 2 and $2/3$ or $\frac{4}{3}$, $-\frac{8}{3}$ and $\frac{16}{3}$.

(c) $V \propto h \propto r^2$

$V = kr^2h$

Find k when $r = 6cm$, $h = 10cm$ and $V = 120\pi cm^3$

3 (c) So, $K = \frac{V}{r^2 h} = \frac{120\pi cm^3}{(6cm)^2 \cdot 10cm}$

$$K = \frac{120}{360}\pi = \frac{1}{3}\pi$$

So, $V = \frac{1}{3}\pi r^2 h$

Req: Volume (V) when $r = 7cm$ and $h = 12cm$

So, $V = \frac{1}{3}\pi (7cm)^2 \cdot 12cm$

$$V = 196\pi\ cm^3$$

$\therefore$ $V = 196\pi cm^3$.

4. (a) Given: $f(x) = 1 - x^3$

Req: $f'(x)$ by 1st principle.

Recall: $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$$f'(x) = \lim_{h \to 0} \frac{1 - (x+h)^3 - [1 - x^3]}{h}$$

$$= \lim_{h \to 0} \frac{1 - [x^3 + 3x^2h + 3xh^2 + h^3] - 1 + x^3}{h}$$

$$= \lim_{h \to 0} \frac{1 - 3x^2h - 3xh^2 + h^3 - 1}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}[-3x^2 - 3xh + h^2]}{\cancel{h}}$$

4 (a) $= \lim_{h \to 0} -3x^2 - 3xh + h^2$

as $h \to 0$; $f'(x) = -3x^2 - 3x(0) + (0)^2$

$\therefore f'(x) = -3x^2$

(b). Given: $x^2 + y^2 - 6xy + 3x - 2y + 5 = 0$

Req: $\frac{dy}{dx} = ?$

$2x + 2y\frac{dy}{dx} - 6\left[x\frac{dy}{dx} + y\right] + 3 - 2\frac{dy}{dx} = 0$

$2x + 2y\frac{dy}{dx} - 6x\frac{dy}{dx} - 6y + 3 - 2\frac{dy}{dx} = 0$

$2y\frac{dy}{dx} - 6x\frac{dy}{dx} - 2\frac{dy}{dx} = 6y - 2x - 3$

$\frac{dy}{dx}\frac{(2y - 6x - 2)}{2y - 6x - 2} = \frac{6y - 2x - 3}{2y - 6x - 2}$

$\therefore \frac{dy}{dx} = \frac{6y - 2x - 3}{2y - 6x - 2}$

(c) Given: $f(x) = y = 2x^3 - 3x^2 - 36x + 14$

Req: stationary points and Their value

$y = 2x^3 - 3x^2 - 36x + 14$

$\frac{dy}{dx} = 6x^2 - 6x - 36$

at stationary point; $\frac{dy}{dx} = 0$

so $0 = 6x^2 - 6x - 36$

4 (c) Solve for x; $x = 3$ or -2

Find y: when $x = 3$; $y = -67$

$x = \cancel{-258}\ -2$, $y = +58$

$\because$ The turning points are $(3, -67)$ and $(-2, 58)$!

Nature: Find $\frac{d^2y}{dx^2}$

So, $\frac{d^2y}{dx^2} = 12x - 6$

For point $(3, -67)$; $\left.\frac{d^2y}{dx^2}\right|_{x=3} = 30 > 0$, then it is minimum point

For point $(-2, 58)$; $\left.\frac{d^2y}{dx^2}\right|_{x=-2} = -30 < 0$, then it is maximum point.

05. (a) Req: Evaluate $\int_1^3 x\sqrt{x^2+1}\,dx$

Let $u = x^2 + 1$

$\frac{du}{dx} = 2x \Rightarrow dx = \frac{du}{2x}$

Change limits

x	u
1	2
3	10

5 (a) So, $\int_1^3 x\sqrt{x^2+1}\,dx = \int_2^{10} \cancel{x} \cdot u^{1/2} \cdot \frac{du}{2\cancel{x}}$

$$= \frac{1}{2}\int_2^{10} u^{\frac{1}{2}}\,du$$

$$= \frac{1}{2}\left[\frac{u^{3/2}}{3/2}\right]_2^{10}$$

$$= \frac{1}{3}\left[u^{3/2}\right]_2^{10}$$

$$= \frac{1}{3}\left[10^{3/2} - 2^{3/2}\right]$$

$$= 9.598 \ (3.d.p)$$

$$\therefore \int_1^3 x\sqrt{x^2+1}\,dx = 9.598.$$

(b). Given: $f'(x) = x^2 - 1$ and $f(3) = 2$

Req: $f(x) = ?$

$$f'(x) = \frac{d}{dx}(f(x)) = x^2 - 1$$

$$\int d(f(x)) = \int (x^2 - 1)\,dx$$

$$f(x) = \frac{x^3}{3} - x + c$$

But $f(3) = 2$: $2 = \frac{(3)^3}{3} - 3 + c$

$$2 = 9 - 3 + c \Rightarrow 2 = 6 + c$$

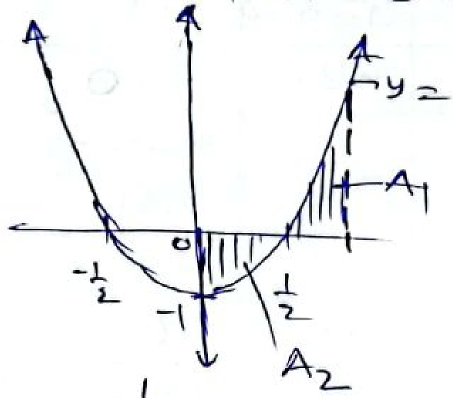
$$c = -4$$

$$\therefore f(x) = \frac{x^3}{3} - x - 4.$$

5 (c). Given: $f(x) = 4x^2 - 1$

Req: Area btn $x = 0$ and $x = 1$

sketch.



Recall: $A = \int_a^b f(x)\,dx$

But $\int_0^1 (4x^2 - 1)\,dx = \int_0^{\frac{1}{2}} (4x^2 - 1)\,dx + \int_{\frac{1}{2}}^1 (4x^2 - 1)\,dx$

$$= \left[\frac{4x^3}{3} - x\right]_0^{\frac{1}{2}} + \left[\frac{4x^3}{3} - x\right]_{\frac{1}{2}}^1$$

$$= \left|\frac{-1}{3} - 0\right| + \left|\frac{1}{3} - \left(-\frac{1}{3}\right)\right|$$

$$= \left|\frac{1}{3}\right| + \left|\frac{2}{3}\right| = 1$$

$\therefore$ Area $= 1$ square unit.

6 (a) Frequency distribution table

$A = 57$

Class interval	f	x	$d = x - A$	fd	c.f
35 – 39	02	37	−20	$-\cancel{8}4\cancel{0}$	02
40 – 44	01	42	−15	~~−630~~ −15	03
45 – 49	04	47	−10	−40	07
50 – 54	02	52	−5	−10	09
55 – 59	04	57	0	0	13
60 – 64	05	62	5	25	18
65 – 69	03	67	10	30	21
70 – 74	00	72	15	0	21
75 – 79	01	77	20	20	22
Σ	22			−30	

(b) Recall: $\bar{x} = A + \dfrac{\Sigma fd}{\Sigma f}$

$$\bar{x} = 57 + \left(\frac{-30}{22}\right)$$

$$= 55.6\cancel{2} \ (1 \text{ d.p})$$

$\therefore \ \bar{x} = 55.6$

(c) Hint: Quartile deviation $= \text{S.I.R} = \dfrac{Q_3 - Q_1}{2}$

Recall: $Q_i = L_{Q_i} + \left(\dfrac{\frac{i}{4}N - f_b}{f_w}\right)c$

For Q_3: Q_3 position $= \frac{3}{4}(22)^{th} = 16.5^{th}$ position

so, Q_3 class interval $= 60 - 64$

then, $f_b = 13$, $f_w = 05$, $L_{Q_3} = 59.5$, $c = 5$

6 (c). So, $Q_3 = 59.5 + \left[\frac{15.5 - 13}{5}\right] \times 5$

$$Q_3 = 63.$$

For Q_1: Q_1 position $= \frac{1}{4}(22)^{th} = 5.5^{th}$ position

So, Q_1 class interval $= 45-49$

then $fb = 03$, $fw = 04$, $LQ_1 = 44.5$, $c = 5$

So, $Q_1 = 44.5 + \left[\frac{5.5 - 3}{4}\right] \times 5$

$$Q_1 = 47.625$$

then $Q \cdot D = \frac{63 - 47.625}{2} \approx 7.69 \text{ (2d.p)}$

$\therefore$ Quartile deviation $= 7.69$ (2 decimal places).

7. (a) Given: $P(B) = 2/3$, $P(A \cap B) = 1/4$

Req: $P(A/B) = ?$

Recall: $P(A/B) = \frac{P(A \cap B)}{P(B)}$

$$P(A/B) = \frac{1/4}{2/3} = 3/8$$

$\therefore P(A/B) = \underline{3/8}$

7 (b). Given: $P(A) = 0.3$, $P(B) = 0.4$

$P(A \cap B) = 0.1$

Req: $P(A' \cap B') = ?$

Hint: $P(A' \cap B') = P(A \cup B)'$

Recall: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cup B) = 0.3 + 0.4 - 0.1$

$P(A \cup B) = 0.6$

also, $P(A \cup B) + P(A \cup B)' = 1$

So, $0.6 + P(A \cup B)' = 1$

$P(A \cup B)' = 0.4$

$\therefore P(A' \cap B') = 0.4$

(c). $n(S) = 20$, $n(G) = 9$, $n(R) = 6$

$n(\text{others}) = 5$.

Req: (i) $P(R') = ?$

Recall: $P(R) + P(R') = 1$

$\frac{n(R)}{n(S)} + P(R') = 1$

$\frac{6}{20} + P(R') = 1$

$\frac{3}{10} + P(R') = 1 \Rightarrow P(R') = \frac{7}{10}$

$\therefore P(R') = \frac{7}{10}$.

7. (c) (ii) ~~How man~~ Let x be the number of red balls added.

Recall: $P(R) = \frac{n(R)}{n(S)}$

$$\frac{1}{2} = \frac{6+x}{20+x}$$

$$12 + 2x = 20 + x$$

$$2x - x = 20 - 12$$

$$x = 8.$$

∴ The 8 red balls should be added.

8 (a) Given: $\frac{\sin 2A + \sin A}{\cos 2A + \cos A + 1} = \tan A$

Req: To prove.

Consider L.H.S

$$\frac{\sin 2A + \sin A}{\cos 2A + \cos A + 1} = \frac{2\sin A\cos A + \sin A}{\cos^2 A - \cancel{\sin^2 A} + \cos A + \cancel{\sin^2 A} + \cos^2 A}$$

$$= \frac{2\sin A\cos A + \sin A}{2\cos^2 A + \cos A}$$

$$= \frac{\sin A(2\cos A + 1)}{\cos A(2\cos A + 1)} = \frac{\sin A}{\cos A}$$

$$= \tan A$$

Since L.H.S = R.H.S #.

CamScanner

8 (b) Given $1+\cos x = 2\sin^2 x$

Reqi angle x from 0 to 2π

$$1+\cos x = 2(1-\cos^2 x)$$

$$1+\cos x = 2-2\cos^2 x$$

$$\Rightarrow 2\cos^2 x + \cos x - 1 = 0$$

solve for $\cos x$; $\cos x = \frac{1}{2}$ or -1

Recall: General solution of $\cos x$

$$x = 360^\circ n \pm \cos^{-1}(c)$$

For $\cos x = \frac{1}{2}$; $x = 2\pi n \pm \cos^{-1}\left(\frac{1}{2}\right)$

$$x = 2\pi n \pm \frac{\pi}{3}$$

$$x = \frac{\pi}{3}, \frac{5\pi}{3},$$

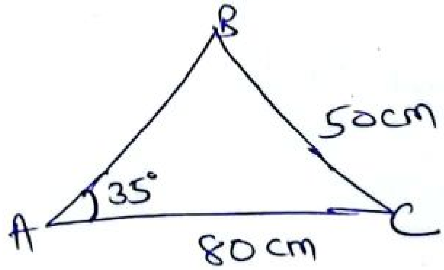
For $\cos x = -1$, $x = 2\pi n \pm \cos^{-1}(-1)$.

$$x = 2\pi n \pm \pi$$

$$x = \pi,$$

$\therefore$ $x = \frac{\pi}{3}$, π and $\frac{5\pi}{3}$.

(c) Consider ΔABC



8. (c)(i) $A\hat{B}C = ?$

Recall sine rule: $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

So, $\frac{\sin 35^\circ}{50cm} = \frac{\sin B}{80cm}$

$$\sin B = \frac{80 \sin 35^\circ}{50}$$

$$\sin B = 0.9177222982$$

$$B = \sin^{-1}(0.9177222982)$$

$$\therefore B = 66.6^\circ$$

$\therefore \angle ABC = 66.6^\circ$ (1 decimal places)

(ii) Side $\overline{AB}$

By using cosine rule

$\cancel{b^2 = a^2 + c^2 - 2ac\cos\hat{B}}$

$\cancel{80^2 = \overline{AB}}$

Recall: $\hat{A} + \hat{B} + \hat{C} = 180^\circ$

$$35^\circ + 66.6^\circ + \hat{C} = 180^\circ$$

$$\hat{C} = 78.4^\circ$$

So, $\frac{\sin 35^\circ}{50} = \frac{\sin 78.4^\circ}{\overline{AB}}$

$\overline{AB} = 85.39$ cm,

$\therefore \overline{AB} = 85.39$ cm,

9 (a) Req: To show that $(\log_x^y)(x\log_y^z) = x\log_x^z$

Consider. L·H·S

$$(\log_x^y)(x\log_y^z) = \frac{\log \cancel{y}}{\log x} \cdot x \cdot \frac{\log z}{\log \cancel{y}} = \frac{x\log z}{\log x}$$

$$= x\log_x^z .$$

Since L·H·S = R·H·S #

(b): Given: $y = {}^{-}\log_2(x+b)$

(i) Req: Determine value of b

use (0, 0); $0 = {}^{-}\log_2(0+b)$

$0 = \log_2 b \Rightarrow b = 2^0 = 1$

$\therefore b = 1.$

(ii) Req: $f^{-1}(x)$.

$f(x) = {}^{-}\log_2(x+1)$

$-y = \log_2(x+1)$

Interchange the variable

$-x = \log_2(y+1)$

Make y the subject

$y+1 = 2^{-x}$

$y = 2^{-x} - 1$

$\therefore f^{-1}(x) = 2^{-x} - 1$

9 (iii) $f^{-1}(x) = 2^{-x} - 1$

$\therefore$ Domain $= \{x : x \in \mathbb{R}\}$

Range: set $2^{-x} = 0$, $y = -1$

$\therefore$ Range $= \{y : y > -1\}$

10 (a) Given $x + y = \begin{pmatrix} 5 & 2 \\ 0 & 9 \end{pmatrix}$ and $x - y = \begin{pmatrix} 3 & 6 \\ 0 & -1 \end{pmatrix}$

Req: Matrix x and y

By elimination method

$$+\left|\begin{array}{l} x + y = \begin{pmatrix} 5 & 2 \\ 0 & 9 \end{pmatrix} \\ x - y = \begin{pmatrix} 3 & 6 \\ 0 & -1 \end{pmatrix} \end{array}\right.$$

$$\frac{2x}{2} = \frac{1}{2}\begin{pmatrix} 8 & 8 \\ 0 & 8 \end{pmatrix} = \begin{pmatrix} 4 & 4 \\ 0 & 4 \end{pmatrix}$$

$$\therefore \quad x = \begin{pmatrix} 4 & 4 \\ 0 & 4 \end{pmatrix}$$

also,

$$-\left|\begin{array}{l} x + y = \begin{pmatrix} 5 & 2 \\ 0 & 9 \end{pmatrix} \\ x - y = \begin{pmatrix} 3 & 6 \\ 0 & -1 \end{pmatrix} \end{array}\right.$$

$$2y = \begin{pmatrix} 2 & -4 \\ 0 & -10 \end{pmatrix} \Rightarrow y = \begin{pmatrix} 1 & -2 \\ 0 & -5 \end{pmatrix}$$

$$\therefore \quad x = \begin{pmatrix} 4 & 4 \\ 0 & 4 \end{pmatrix} \text{ and } y = \begin{pmatrix} 1 & -2 \\ 0 & -5 \end{pmatrix}.$$

10 (b) Let number ~~of shirts~~ produced be x
~~number~~ of pants produced be y

Let, Number of items produced by tailor A be x
Number of items produced by tailor B be y

Minimize; $f(x,y) = 9250x + 12000y$

subjected to; $9x + 15y \geq 90$

$6x + 6y \geq 48$

$x \geq 0$

$y \geq 0$

Table of ~~corner~~ intercepts points

$9x + 15y = 90$

x	0	10
y	6	0

$6x + 6y = 48$

x	0	8
y	8	0

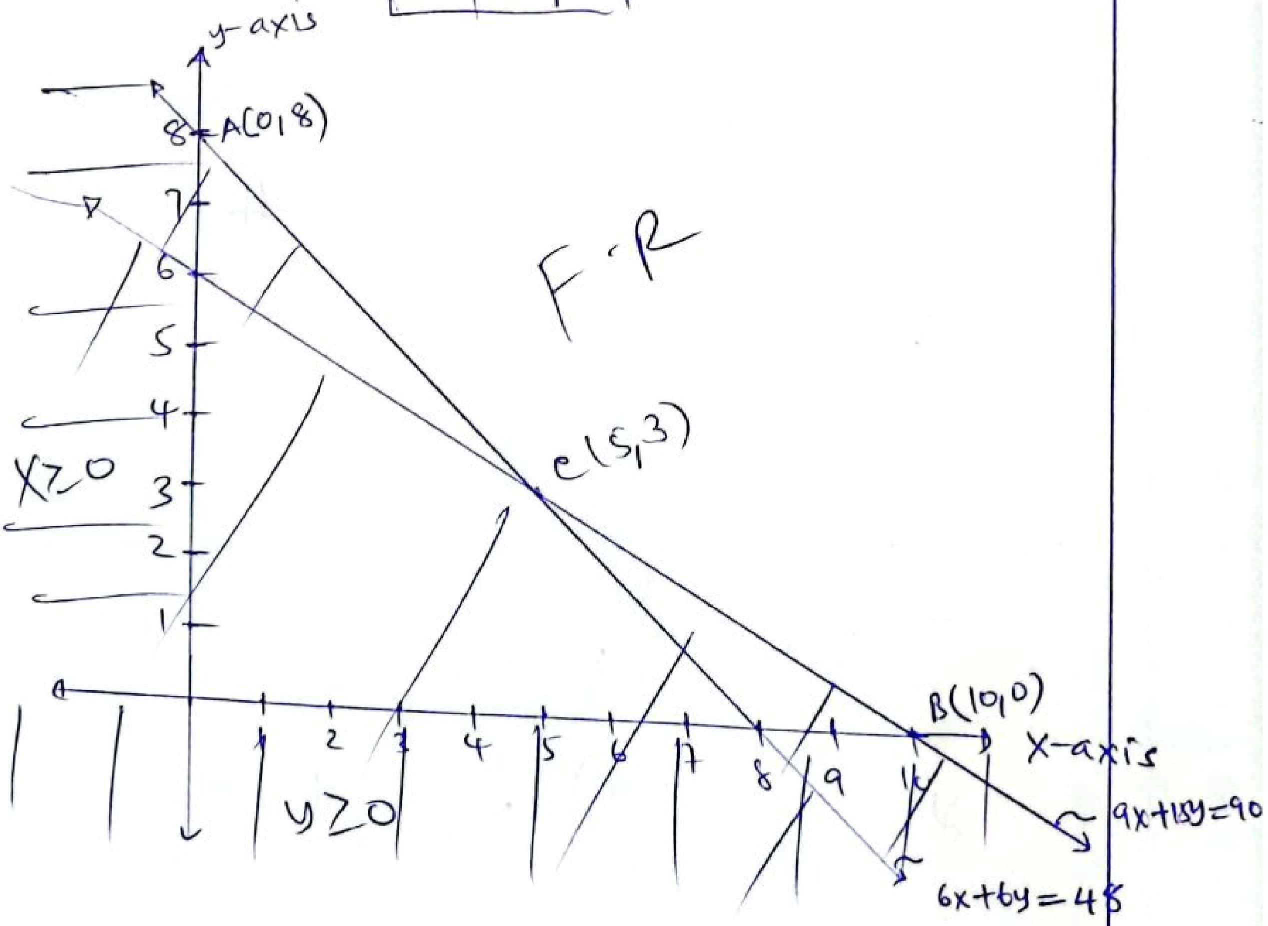


Table of corner points

Corner points	$f(x,y) = 9250x + 12000y$
$A(0,8)$	96,000/=
$B(10,0)$	92,500/=
$C(5,3)$	82250/=

$\therefore$ For minimum labour cost, tailor A should produce 5 items and tailor B should produce 3 items.