07 c) Soln:

Using the differential equation

$$\frac{dN}{dt} = -kN$$

let the initial mass be N_0

From $\frac{dN}{dt} = -kN$

$$\cancel{\frac{dN}{dN}} - k\int dt$$

$$\frac{dN}{N} = -k\,dt$$

Integrate both sides as follows

$$\int \frac{dN}{N} = -k\int dt$$ c1 mark

Solving the integral gives

$\ln|N| = -kt + c$, where c is an arbitrary constant. It follows that

$$N = e^{-kt+c}$$

$$N = Ae^{-kt}, \text{ where } e^{c} = A$$ c1 mark

At $t = 0$, $N = N_0$ (Substituting these values into $N = Ae^{-kt}$ gives,

$$N_0 = Ae^{0} \Rightarrow A = N_0$$

Thus, the required model equation is $N = N_0 e^{-kt}$

Now, at time $t = 8$ days, $N = \frac{N_0}{2}$ c1 mark

Substitute these values into the model equation to obtain value of k

$$\frac{N_0}{2} = N_0 e^{-k(8)}$$

$$k = \frac{1}{8}\ln 2$$ c1 mark

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1(c)(i) from $\int_a^b f(x)dx = 1$

$$\int_0^4 Kx\,dx = 1 \quad \text{c1 mark}$$

$$\int_0^2 Kx\,dx + \int_2^4 K(4-x)\,dx = 1 \quad \text{c1 mark}$$

$$\left[\frac{Kx^2}{2}\right]_0^2 + \left[K\left(4x - \frac{x^2}{2}\right)\right]_2^4 = 1$$

$$2K + 2K = 1$$

$$\frac{4K}{4} = \frac{1}{4} \quad \text{c1 mark}$$

$$K = 1/4$$

(ii) $P\left(\frac{1}{2} \leq x \leq 5/2\right) = \frac{15}{32} + \frac{7}{32}$ c1 mark

$$= \frac{22}{32}$$

$$= 11/16 \quad \text{c1 mark}$$

3 c) ii)

The workdone by force

$W \cdot d = F \cdot d$

$$W \cdot d = \frac{1}{7}(39i + 4j + 33k) \cdot (2i - 1j - 2k)$$

$$W \cdot d = \frac{1}{7}[(39)(2) + (4)(-1) + (33)(-2)]$$

$$W \cdot d = \frac{1}{7}(78 + {}^{-}16 + {}^{-}66)$$

$$W \cdot d = -\frac{4}{7}$$

$$= \left|-4/7\right| = 4/7$$

$\therefore$ The workdone by the force is $+\frac{4}{7}$ units

0/1 mark

08

(c)

i)

Given $r = \dfrac{4}{3+5\sin\theta}$

from $y = r\sin\theta \Rightarrow \sin\theta = y/r$

$$r = \frac{4}{3+5\left[\frac{y}{r}\right]}$$

01 mark

$$r = \frac{4r}{3r+5y}$$

Dividing by r both sides gives

$$1 = \frac{4}{3r+5y}$$

01 mark

$\Rightarrow 3r + 5y = 4$

$3r = 4 - 5y$

Squaring both sides results into

$9r^2 = 16 - 40y + 25y^2$

But $r^2 = x^2 + y^2$.

$9x^2 - 16y^2 + 40y - 16 = 0$

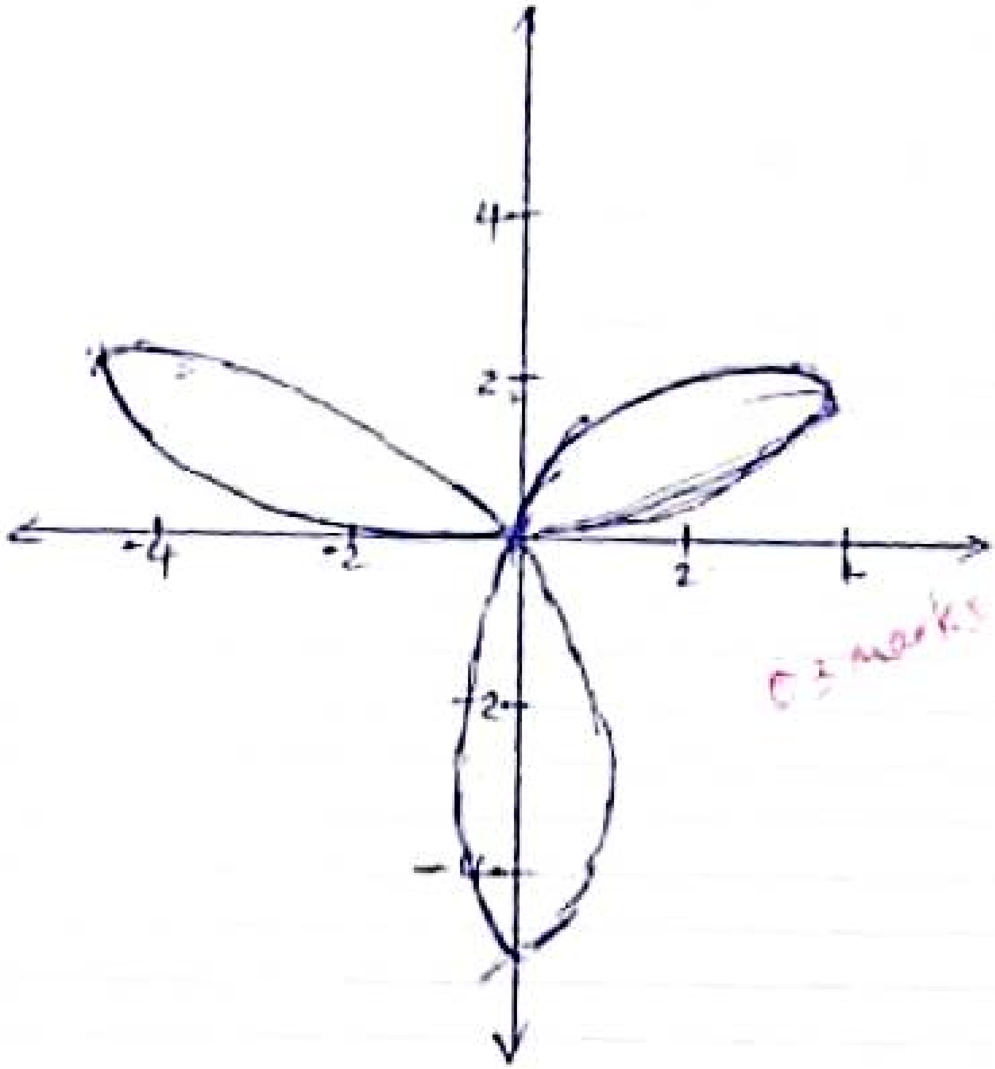
01½ mark

Therefore the rectangular equation is

$\therefore 9x^2 - 16y^2 - 40y - 16 = 0$.

≈ 3½

A graph of $r = 5\sin 3\theta$



(2 marks)

c)

Substitute the value of k, t = 30 days and initial value $N_0 = 200g$ into the model equation to get

$$N = 200e^{-(\frac{1}{3}\ln 2)30}$$

$$= 14.865g$$

Therefore, the mass of the radioactive substance after 30 days is $14.865g$

5(a) $\dfrac{(x+1)(x-3)(x+4)}{x-2} \leq 0$

Let $x+1=0$, $x-3=0$, $x+4=0$, $x-2=0$

$x = -1, 3, -4, 2$ 0½ mark

$y = \dfrac{(x+1)(x-3)(x+4)}{x-2}$

	$x \leq -4$	$-4 \leq x \leq -1$	$-1 \leq x < 2$	$2 < x \leq 3$	$x \geq 3$
$x+1$	-ve	0 or -ve	0 or +ve	+ve	+ve
$x-3$	-ve	-ve	-ve	0 or -ve	0 or +ve
$x+4$	0 or -ve	0 or +ve	+ve	+ve	+ve
$x-2$	-ve	-ve	-ve	+ve	+ve
$(x+1)(x-3)(x+4)$	-ve	+ve	-ve	-ve	+ve
y	+ve	-ve	+ve	-ve	+ve
	0½	0½	00½	01	01

From the table above

The Solution is $-4 \leq x \leq -1$ or $2 < x \leq 3$ 01 mark

Note: 2 is not included because once the denominator is zero then the inequality will be undefined

8. (a) $y = mx + c$, $4x^2 + 9y^2 = 36$

$$y^2 = (mx + c)^2$$

$$y^2 = m^2x^2 + 2mcx + c^2$$ 01 mark

$$4x^2 + 9(m^2x^2 + 2mcx + c^2) - 36 = 0$$

$$4x^2 + 9m^2x^2 + 18mcx + 9c^2 - 36 = 0$$

$$(4+9m^2)x^2 + 18mcx + 9c^2 - 36 = 0$$ 01 mark

for tangent:

$$b^2 = 4ac$$

$$(18mc)^2 = 4(4+9m^2)(9c^2 - 36)$$ 01 mark

$$324m^2c^2 = 4(4+9m^2)(9c^2 - 36)$$

$$\frac{324m^2c^2}{36} = \frac{4 \times 4(4+9m^2)(c^2 - 4)}{36}$$

$$9m^2c^2 = (4+9m^2)(c^2 - 4)$$ 01 mark

$$9m^2c^2 = 4c^2 - 16 + 9m^2c^2 - 36m^2$$

$$\frac{4c^2}{4} - \frac{16}{4} - \frac{36m^2}{4} = \frac{0}{4}$$

$$c^2 - 4 - 9m^2 = 0$$

$$c^2 = 9m^2 + 4$$

$$\therefore c = \pm\sqrt{4 + 9m^2}$$ 01 mark

6(d) Recall $(1+x)^n = 1 + nx + \frac{n(n-1)x^2}{2!} + \frac{n(n-1)(n-2)x^3}{3!} + \dots$ — 01 mark

$\sqrt{1+x} = (1+x)^{\frac{1}{2}}$

$(1+x)^{\frac{1}{2}} = 1 + \frac{1}{2}x + \frac{\frac{1}{2}(\frac{1}{2}-1)x^2}{2!} + \frac{\frac{1}{2}(\frac{1}{2}-1)(\frac{1}{2}-2)x^3}{3!} + \dots$ 01 mark

$(1+x)^{\frac{1}{2}} = 1 + \frac{1}{2}x - \frac{1}{8}x^2 + \frac{1}{16}x^3 + \dots$

$(1+x)^{\frac{1}{2}} = 1 + \frac{1}{2}x - \frac{1}{8}x^2$ — 01 mark

$\sqrt{30} = \sqrt{25+5}$

$= \sqrt{25(1+\frac{1}{5})}$

$= 5\sqrt{(1+\frac{1}{5})}$

$= 5\left(1 + \frac{1}{2} \times \frac{1}{5} - \frac{1}{8}(\frac{1}{5})^2\right)$ — 01 mark

$= 5 \times 1.095$

$= 5.475$

$\therefore \sqrt{30} \simeq 5.475$ Correct to 4 Significant figures — 01 mark

4(b) $z^3 = 8i$

$$8i = 8e^{i\pi/2}$$

$|8i| = 8$ and $\arg(8i) = \pi/2$ — 01 mark

now

$$z = \sqrt[3]{8}\; e^{i\frac{1}{3}(\pi/2 + 2k\pi)}$$

$$z = 2e^{i\pi/6 + i\frac{2k\pi}{3}}$$

for $k = 0, 1, 2$

— 01 mark

$$z_0 = 2\left(\cos\pi/6 + i\sin\pi/6\right) = \sqrt{3} + i$$

$$z_1 = 2\left(\cos 5\pi/6 + i\sin 5\pi/6\right) = -\sqrt{3} + i$$

$$z_2 = 2\left(\cos\tfrac{3\pi}{2} + i\sin\tfrac{3\pi}{2}\right) = -2i$$

— 01 mark

$\therefore$ The roots are $\sqrt{3}+i$, $-\sqrt{3}+i$ and $-2i$

2(a) A truth table for $\sim(p \wedge q) \cdot (p \cdot q)$ where $\cdot$ means $\wedge$

p	q	$p \cdot q$	$\sim(p \cdot q)$	$\sim(p \wedge q) \cdot (p \cdot q)$
T	T	T	F	F
T	F	F	T	F
F	T	F	T	F
F	F	F	T	F

01 01 01

The statement is a contradiction

(b) $(\sim p \wedge \sim q) \vee (\sim p \wedge q) \vee (p \cdot q)$

$= \sim p \wedge (\sim q \vee q) \vee (p \cdot q)$ Distributive law

$= (\sim p \wedge T) \vee (p \cdot q)$ $\quad \sim x \vee x = T$ — 01 mark

$= \sim p \vee (p \cdot q)$ $\quad \sim x \wedge T = \sim x$ — 01 mark

$= (\sim p \vee p) \wedge (\sim p \vee q)$ Distributive law

$= T \wedge (\sim p \vee q)$ $\quad \sim x \vee x = T$ — 01 mark

$= \sim p \vee q$ $\quad T \wedge x = x$ — 01 mark

$\therefore (\sim p \wedge \sim q) \vee (\sim p \wedge q) \vee (p \cdot q) = \sim p \vee q$

6(b) Since the power of numerator is greater than that of the denominator, then we divide first

$$\frac{x^3-3x+1}{x^2+4x-12} = x-4+\frac{25x-47}{(x-2)(x+6)}$$

— 01 mark

Consider $\frac{25x-47}{(x-2)(x+6)}$

$$\frac{25x-47}{(x-2)(x+6)} = \frac{A}{x-2} + \frac{B}{x+6}$$ — 01 mark

$$\frac{25x-47}{(x-2)(x+6)} = \frac{A(x+6)+B(x-2)}{(x-2)(x+6)}$$

$$25x-47 = A(x+6)+B(x-2)$$ — 01 mark

Set $x=-6$

$$+197 = +8B$$

$$B = \frac{197}{8}$$

Set $x=2$

01 mark

$$3 = 8A$$

$$A = 3/8$$

$$\therefore \frac{x^3-3x+1}{x^2+4x-12} = x-4+\frac{3}{8(x-2)}+\frac{197}{8(x+6)}$$

— 01 mark

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07 a) i) $\frac{dy}{dx} = yx^2 + 3 + y + 3x^2$

Soln.

Given $\frac{dy}{dx} = yx^2 + 3 + y + 3x^2$

Rearrangement and factorization results into.

$\frac{dy}{dx} = (y+3)(x^2+1)$ 01 mark

Multiply both sides of the equation by dx to obtain $dy = (y+3)(x^2+1)\,dx$

Separating the variables gives,

$\frac{dy}{y+3} = (x^2+1)dx$

Integrate both sides as follows

$\int \frac{dy}{y+3} = \int (x^2+1)\,dx$

$\Rightarrow \ln|y+3| = \frac{x^3}{3} + x + c$ 01 mark

$\Rightarrow y + 3 = e^{\frac{x^3}{3}+x+c}$

But $e^{\frac{x^3}{3}+x+c} = e^{\frac{x^3}{3}+x} \times e^c$.

Where

$e^c = A$ is a constant

Thus.

$y = Ae^{\frac{1}{3}x^3+x} - 3$

Therefore general solution is

$y = Ae^{\frac{1}{3}x^3+x} - 3$ $00\frac{1}{2}$ mark

~ 25 ~

1(b) Soln

$P(WW) = \frac{{}^{7}C_2}{{}^{11}C_2}$ 01 mark

and $P(BB) = \frac{{}^{4}C_2}{{}^{11}C_2}$ 01 mark

$P(\text{of the Same Colour}) = P(WW) + P(BB)$ 01 mark

$= \frac{{}^{7}C_2}{{}^{11}C_2} + \frac{{}^{4}C_2}{{}^{11}C_2}$ 01 mark

$= \frac{21}{55} + \frac{6}{55}$

$= \frac{27}{55}$ 01 mark

CamScanner

(b)(i) $3\sin^2\theta - \sin\theta\cos\theta - 4\cos^2\theta = 0$

Divide both sides by $\cos^2\theta$

$$\frac{3\sin^2\theta}{\cos^2\theta} - \frac{\sin\theta\cos\theta}{\cos^2\theta} - \frac{4\cos^2\theta}{\cos^2\theta} = \frac{0}{\cos^2\theta}$$

$3\tan^2\theta - \tan\theta - 4 = 0$ — 01 mark

Let $t = \tan\theta$

$3t^2 - t - 4 = 0$

By Calculator

$t = 4/3$ or -1

For $t = 4/3$

$\tan\theta = 4/3$ — 01 mark

$\tan\theta$ is +ve in the first and 3rd quadrants

So $\theta = 53.13°, 233.13°$

For $t = -1$

$\tan\theta = -1$

$\tan\theta$ is negative in the second and fourth quadrants

$\theta = 135°, 315°$ — 01 mark

$\therefore \theta = 53.13°, 135°, 233.13°$ and $315°$

5(a)(i) Consider LHS

$$= \frac{\sin A}{\sec A + \tan A - 1} + \frac{\cos A}{\csc A + \cot A - 1}$$

$$= \frac{\sin A}{\frac{1}{\cos A} + \frac{\sin A}{\cos A} - 1} + \frac{\cos A}{\frac{1}{\sin A} + \frac{\cos A}{\sin A} - 1}$$ — 01 mark

$$= \frac{\sin A}{\frac{1 + \sin A - \cos A}{\cos A}} + \frac{\cos A}{\frac{1 + \cos A - \sin A}{\sin A}}$$

$$= \frac{\sin A \cos A}{1 + \sin A - \cos A} + \frac{\sin A \cos A}{1 + \cos A - \sin A}$$ — 01 mark

$$= \sin A \cos A \left(\frac{1}{1 + \sin A - \cos A} + \frac{1}{1 + \cos A - \sin A} \right)$$

$$= \sin A \cos A \left(\frac{1 + \cos A - \sin A + 1 + \sin A - \cos A}{(1 + \sin A - \cos A)(1 + \cos A - \sin A)} \right)$$

$$= \sin A \cos A \left(\frac{2}{(1 + \sin A - \cos A)(1 + \cos A - \sin A)} \right)$$ — 01 mark

But $(1 + \sin A - \cos A)(1 + \cos A - \sin A) = 2 \sin A \cos A$

$$= \frac{2 \sin A \cos A}{2 \sin A \cos A}$$ — 01 mark

$$= 1$$

Since LHS = RHS hence proved

4(i) equating i and ii

$$\frac{1}{\sqrt{3}}(x+1) = -\sqrt{3}(x-1)$$

$$x+1 = -3(x-1)$$ oo $\frac{1}{2}$ mark

$$x+1 = -3x+3$$

$$4x = 2$$

$$x = 1/2$$

$$y = \frac{1}{\sqrt{3}}(1/2+1)$$

$$= \frac{1}{\sqrt{3}} \times 3/2$$ oo $\frac{1}{2}$ mark

$$= \frac{\sqrt{3}}{2}$$

$$\therefore \quad z = 1/2 + \frac{\sqrt{3}}{2} i$$

~ 11 ~

3 b)

$6t^3 - 17t^2 - 604t - 672 = 0$ — 01 mark

$(t-12)(t+8)(6t+7) = 0$ — 01 mark

$\therefore\ t = 12,\ t = -8$ and $t = -7/6$ 01 mark

c) i)

Given

$a = (1, -2, 1)$ and $b = (4, -4, 7)$

$$Proj_b\, a = \frac{a \cdot b}{|b|^2}\, b$$ 01 mark

$$Proj_b\, a = \frac{(i - 2j + k)\cdot(4i - 4j + 7k)}{|4i - 4j + 7k|^2}(4i - 4j + 7k)$$

$$= \frac{4 + 8 + 7}{(\sqrt{81})^2}(4i - 4j + 7k)$$ 01 mark

$$= \frac{19}{81}(4i - 4j + 7k)$$

$$= \frac{76}{81}i - \frac{76}{81}j + \frac{133}{81}k$$ 01 mark

The Vector projection of a onto b is $\frac{76}{81}i - \frac{76}{81}j + \frac{133}{81}k$

~ C7 ~

Pre multiply the inverse of R both sides of matrix equation

$$\begin{pmatrix} \frac{1}{50} & \frac{1}{10} & \frac{13}{50} \\ 4/25 & -1/5 & 7/25 \\ 11/50 & 1/5 & -7/50 \end{pmatrix}\begin{pmatrix} 1 & 2 & 3 \\ 2 & -3 & 2 \\ 3 & 1 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \frac{1}{50} & \frac{1}{10} & \frac{13}{50} \\ 4/25 & -1/5 & 7/25 \\ 11/50 & 1/5 & -7/50 \end{pmatrix}\begin{pmatrix} 6 \\ 14 \\ -2 \end{pmatrix}$$

01 mark

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1/50 \times 6 + \frac{1}{10} \times 14 + 13/50 \times -2 \\ 4/25 \times 6 + \frac{-1}{5} \times 14 + 7/25 \times -2 \\ 11/50 \times 6 + \frac{1}{5} \times 14 + -7/50 \times -2 \end{pmatrix}$$

01 mark

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$$

$\therefore$ $x = 1$, $y = -2$ and $z = 1$ 01 mark

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01 (a) (i)

Position	1^{st}	2^{nd}	3^{rd}	4^{th}	5^{th}
Possibilities	2	4	3	2	1

c1 mark

Total numbers $= 2 \times 4 \times 3 \times 2 \times 1$

$= 48 = 48$ c1 mark

$\therefore$ 48 numbers greater than 40,000 can be formed

(ii) Number of ways $= 4 \times 4 \times 4 = 4^3$ c1 mark

$= 64$

$\therefore$ 64 ways c1 mark

08. (a).

$r = 5 \sin 3\theta$

Table of values

θ	$5 \sin 3\theta$	(r, θ)
0°	0	(0,0)
30°	5	(5, 30°)
60°	0	(0, 60°)
90°	-5	(-5, 90°)
120°	0	(0, 120°)
150°	5	(5, 150°)
180°	0	(0, 180°)
210°	-5	(-5, 210°)
240°	0	(0, 240°)
270°	5	(5, 270°)
300°	0	(0, 300°)
330°	-5	(-5, 330°)
360°	0	(0, 360°

02 marks

4 (a) Let $z_1 = a+bi$ and $z_2 = c+di$

$\bar{z}_1 + \bar{z}_2 = (a-bi) + (c-di)$ — 01 mark

$= (a+c) - (b+d)i$

again $z_1 + z_2 = a+bi+c+di$

$= (a+c) + (b+d)i$ — 01 mark

$\overline{z_1+z_2} = (a+c) - (b+d)i$ — 01 mark

Since RHS = LHS hence shown

(1) Let $z = x+iy$
then

$z+1 = (x+1)+iy$

$\arg(z+1) = \tan^{-1}\left(\frac{y}{x+1}\right) = \pi/6$

$z-1 = (x-1)+iy$ 01 mark

$\arg(z-1) = \tan^{-1}\left(\frac{y}{x-1}\right) = \frac{2\pi}{3}$

$\frac{y}{x+1} = \tan\pi/6 = \frac{1}{\sqrt{3}}$

also

$\frac{y}{x-1} = \tan\frac{2\pi}{3} = -\sqrt{3}$ — 01 mark

$y = -\sqrt{3}(x-1)$ --- i

$y = \frac{1}{\sqrt{3}}(x+1$ --- ii

4 ii $9z^4 - 35z^2 - 4.2 = 0$

Let $z^2 = a$

$9a^2 - 35a - 4.2 = 0$

01 mark

$$a = +\frac{+35 \pm \sqrt{(-35)^2 - (4 \times 9 \times -4.2)}}{2 \times 9}$$

$$a = \frac{35 \pm \sqrt{1225 + 151.2}}{18}$$

$$a = \frac{35 \pm 37.097}{18}$$

01 mark

$a = 4.005$ or -0.1165

but $a = z^2$

when $a = 4.005$

$z = \pm\sqrt{4.005}$

$z = \pm 2.001$

when $a = -0.1165$

$z = \pm i\sqrt{0.1165}$

$z = \pm 0.341i$

01 mark

$\therefore z = 2.001, -2.001, 0.341i, -0.341i$

2 c) Sol

P	Q	R	S	Basic conjunction
T	T	T	T	$P \land Q \land R$
T	T	F	T	
T	F	T	F	
T	F	F	F	
F	T	T	T	$\sim P \land Q \land R$
F	T	F	F	
F	F	T	F	
F	F	F	T	$\sim P \land \sim Q \land \sim R$

0.5 marks

∴ The compound statement for the letters is $(P \land Q \land R) \lor (\sim P \land Q \land R) \lor (\sim P \land \sim Q \land \sim R)$. 1 mark

d)

Solution

Let p: Halima reaches home early
q: The traffic jam is there. 1 mark

The premises and conclusion of the argument can be written $p \lor \sim q ; \sim q ; \therefore \sim p$

$[(p \lor \sim q) \land \sim q] \to \sim p$ 0.1 mark

the truth table:

p	q	~p	~q	$p \lor \sim q$	$(p \lor \sim q) \land \sim q$	$((p \lor \sim q) \land \sim q) \to \sim p$
T	T	F	F	T	F	T
T	F	F	T	T	T	F
F	T	T	F	F	F	T
F	F	T	T	T	T	T

1 mark

1 mark

∴ The truth values in the last column are not all true. Therefore, the argument is Not Valid 0.5

08. (c)

(ii)

$$(x^2+y^2)^3 = xy(x^2-y^2)$$

from $y = r\sin\theta$, $x = r\cos\theta$ and

$$x^2+y^2 = r^2.$$

01 mark

Substituting into equation results into

$$(r^2)^3 = r\cos\theta \cdot r\sin\theta(r^2\cos^2\theta - r^2\sin^2\theta)$$

Simplification

$$r^6 = r^4\sin\theta\cos\theta(\cos^2\theta - \sin^2\theta) \quad \text{---(i)}$$

But:

$$\sin\theta\cos\theta = \tfrac{1}{2}\sin 2\theta \quad \text{---- (ii)}$$

$$\cos^2\theta - \sin^2\theta = \cos 2\theta \quad \text{---- (iii)}$$

Substituting equation (ii), (iii) into equation (i)

$$r^6 = \tfrac{1}{2}r^4\sin 2\theta\cos 2\theta$$

01 mark

Also: $\sin 2\theta\cos 2\theta = \tfrac{1}{2}\sin 4\theta$.

$$\Rightarrow r^6 = \tfrac{1}{4}r^4\sin 4\theta$$

Dividing by r^4 both sides

$$r^2 = \tfrac{1}{4}\sin 4\theta.$$

00½ mark

$\therefore$ Therefore the polar equation is

$$r^2 = \tfrac{1}{4}\sin 4\theta$$

~ 3mks

$$= 2\sin\frac{C}{2}\left(\cos\frac{A-B}{2} - \sin\frac{C}{2}\right)$$

But $\sin\frac{C}{2} = \cos\frac{A+B}{2}$ — 01 mark

$$= 2\sin\frac{C}{2}\left(\cos\frac{A-B}{2} - \cos\frac{A+B}{2}\right)$$

$$= 2\sin\frac{C}{2}(-2)\sin\left(\frac{\frac{A-B}{2}+\frac{A+B}{2}}{2}\right)\sin\left(\frac{\frac{A-B}{2}-\frac{A+B}{2}}{2}\right)$$ — 01 mark

$$= -4\sin\frac{C}{2}\sin\frac{A}{2}\sin\frac{-B}{2}$$

$$= -4 \times -1 \sin\frac{C}{2}\sin\frac{A}{2}\sin\frac{B}{2}$$

$$= 4\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$$ — 01 mark

Since LHS = RHS hence proved

4 (iii) $\arg\left(\frac{z-1}{z+1}\right) = \pi/4$

$$\arg\left(\frac{x+iy-1}{x+iy+1}\right) = \pi/4$$

$$\arg\left(\frac{(x-1)+iy}{(x+1)+iy}\right) = \pi/4$$ — 01 mark

$$\arg\left(\frac{(x-1)+iy}{(x+1)+iy}\cdot\frac{(x+1)-iy}{(x+1)-iy}\right) = \pi/4$$

$$\arg\left(\frac{x^2-1-(xy-y)i+(xy+y)i+y^2}{(x+1)^2+y^2}\right) = \pi/4$$

—01 mark

$$\arg\left(\frac{x^2-1-xyi+yi+xyi+yi+y^2}{(x+1)^2+y^2}\right) = \pi/4$$

$$\arg\left(\frac{x^2+y^2-1+2yi}{(x+1)^2+y^2}\right) = \pi/4$$

$$\tan^{-1}\left(\frac{2y}{x^2+y^2-1}\right) = \pi/4$$

$$\frac{2y}{x^2+y^2-1} = \tan \pi/4$$

01 mark

$$\frac{2y}{x^2+y^2-1} = 1$$

$$x^2+y^2-2y-1=0$$

The locus is a circle with centre (0, 1)

5 b(ii) Let $12\cos\theta + 5\sin\theta = R\sin(\theta+\alpha)$

$12\cos\theta + 5\sin\theta = R\sin\theta\cos\alpha + R\cos\theta\sin\alpha$

$R = \sqrt{a^2+b^2}$ — 01 mark

$R = \sqrt{12^2+5^2}$

$R = \sqrt{144+25}$

$R = 13$

Recall $-1 \le \sin\theta \le 1$ — 01 mark

$-1 \le \sin(\theta+\alpha) \le 1$

$-R \le R\sin(\theta+\alpha) \le R$

$-13 \le 13\sin(\theta+\alpha) \le 13$ — 01 mark

$\therefore$ The minimum value is -13 and the maximum value is 13

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5(c) Consider LHS

By sine rule

$b = K\sin B$

$c = K\sin C$ — 01 mark

$$\frac{b-c}{b+c} = \frac{K\sin B - K\sin C}{K\sin B + K\sin C}$$ — 01 mark

$$= \frac{K(\sin B - \sin C)}{K(\sin B + \sin C)}$$ — 01 mark

Using factor formula

$$= \frac{2\cos\frac{B+C}{2}\sin\frac{B-C}{2}}{2\sin\frac{B+C}{2}\cos\frac{B-C}{2}}$$ — 01 mark

$$= \cot\frac{B+C}{2}\tan\frac{B-C}{2}$$ — 01 mark

$$= \tan\frac{B-C}{2}\cot\frac{B+C}{2}$$ — 01 mark

Since LHS = RHS hence proved

From

3 (c) ii) $F_1 = f_m\left(\frac{f}{|f|}\right)$

$F_1 = 5\left[\frac{6i + 2j + 3k}{\sqrt{6^2 + 2^2 + 3^2}}\right]$ — 01 mark

$F_1 = \frac{5}{7}(6i + 2j + 3k)$

$F_2 = f_m\left[\frac{f}{|f|}\right]$

$F_2 = 3\left[\frac{3i - 2j + 6k}{\sqrt{3^2 + (-2)^2 + 6^2}}\right]$ — 01 mark

$F_2 = \frac{3}{7}(3i - 2j + 6k)$

The Resultant force is given by

$F = F_1 + F_2$

$= \frac{5}{7}(6i + 2j + 3k) + \frac{3}{7}(3i - 2j + 6k)$

$F = \frac{1}{7}(39i + 4j + 33k)$

The displacement vector d is given by

$d = \vec{AB} = (4i - 3k + k) - (2i + j + 3k)$

$d = (4-2)i + (-3-1)j + (1-3)k$

$d = 2i - 4j - 2k$ — 01 mark

6(c)(i) $R = \begin{pmatrix} 1 & 2 & 3 \\ 2 & -3 & 2 \\ 3 & 1 & -1 \end{pmatrix}$

$$|R| = 1(1) - 2(-8) + 3(11)$$

$$|R| = 50$$ — 01 mark

Matrix of Cofactors of $R = \begin{pmatrix} 1 & 8 & 11 \\ 5 & -10 & 5 \\ 13 & 4 & -7 \end{pmatrix}$

$$\text{Adj } R = \begin{pmatrix} 1 & 5 & 13 \\ 8 & -10 & 4 \\ 11 & 5 & -7 \end{pmatrix}$$ — 0 0$\frac{1}{2}$ mark

$$R^{-1} = \frac{1}{|R|} \text{adj} R = \frac{1}{50} \begin{pmatrix} 1 & 5 & 13 \\ 8 & -10 & 4 \\ 11 & 5 & -7 \end{pmatrix}$$

$$R^{-1} = \begin{pmatrix} 1/50 & 1/10 & 13/50 \\ 4/25 & -1/5 & 2/25 \\ 11/50 & 1/5 & -7/50 \end{pmatrix}$$ — 00$\frac{1}{2}$ mark

(ii)
$$x + 2y + 3z = 6$$
$$2x - 3y + 2z = 14$$
$$3x + y - z = -2$$

In matrix form

$$\begin{pmatrix} 1 & 2 & 3 \\ 2 & -3 & 2 \\ 3 & 1 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 6 \\ 14 \\ -2 \end{pmatrix}$$

7 a) ii)

Then $y = x\sinh(\ln|x|)$

But $\sinh(\ln x) = \frac{x^2-1}{2x}$

$$y = x\left[\frac{x^2-1}{2x}\right]$$

$$= \frac{1}{2}(x^2-1)$$

only ½ mark

$$\therefore\ y = \frac{1}{2}(x^2-1)$$

b) Given

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} - 6y = 0$$

Corresponding characteristic equation is given by $\lambda^2+\lambda-6=0$ (2 marks)

Factorising the equation

$$(\lambda-2)(\lambda+3)=0$$

The roots are $\lambda=2$ and $\lambda=-3$ (2 marks)

$$y_1 = e^{2x} \text{ and } y_2 = e^{-3x}$$

$\therefore$ The general solution is (1 mark)

$$y = C_1e^{2x} + C_2e^{-3x}$$

= 27 =

08 (i)

$$9y^2 - 16x^2 + 32x - 36y - 124 = 0$$

By completing the square

$$\frac{9}{144}(y^2 - 4y + 4) + \frac{16(x-1)^2}{144} = \frac{144}{144}$$

$$\frac{(y-2)^2}{16} - \frac{(x-1)^2}{9} = 1$$ 01 mark

(i) The required equation is

$$\frac{(y-2)^2}{16} - \frac{(x-1)^2}{9} = 1$$ 01 mark

ii) Compare the equation with

$$\frac{(y-k)^2}{b^2} - \frac{(x-h)^2}{a^2} = 1$$

$a = 3$, $b = 4$, $h = 1$, $k = 2$.

But $a^2 = b^2(e^2 - 1)$

$$\begin{aligned}(be)^2 &= a^2 + b^2 \\ &= 3^2 + 4^2 \\ &= 9 + 16 \\ &= 25\end{aligned}$$ 01 mark

$$\begin{aligned}be &= \pm\sqrt{25} \\ &= \pm 5\end{aligned}$$

(ii) Centre $= (h, k) = (1, 2)$ 01 mark

Foci are $(h, k \pm be) = (1, 2 \pm 5)$

$= (1, 7)$ and $(1, -3)$

$\therefore$ The foci are $(1, 7)$ and $(1, -3)$. 01 mark

=3.5=

3 a)

Given

a) $\underline{a} = 3i - 2j + 3k$

$\underline{b} = 4i + 3j - 4k$

$m : n = 1 : 4$

$\overline{AB} = \left[\frac{m}{m+n}\right] b + \left[\frac{n}{m+n}\right] \underline{a}$ — 01 mark

$\overline{AB} = \left[\frac{1}{1+4}\right] 4i + 3j - 4k + \left[\frac{4}{1+4}\right] 3i - 2j + 3k$ — 01 mark

$\overline{AB} = \frac{4i + 3j - 4k}{5} + \frac{12i - 8j + 12k}{5}$

$\overline{AB} = \frac{16i - 5j + 8k}{5}$ — 01 mark

$\overline{AB} = \frac{16}{5} i - j + \frac{8}{5} k$

$\therefore$ The Required position Vector is

$\overline{AB} = \frac{16}{5} i - j + \frac{8}{5} k$ — 01 mark

b)

Given

$u = 3t^2 i - j - 4tk$

$v = 2ti + 17t^2 j + 151k$

$u \cdot v = \cancel{676}\ 672.$

Then — 01 mark

$u \cdot v = (3t^2 i - j - 4tk) \cdot (2ti + 17t^2 j + 151k)$

$672 = 6t^3 - 17t^2 - 604t$