CHRISTIAN SOCIAL SERVICES COMMISSION

CSSC – SOUTHERN ZONE, FORM FOUR JOINT EXAMINATION 2024

MARKING GUIDE

041 BASIC MATHEMATICS

1\. (a) Let x =Total number of people in the village

Therefore,

$\frac{1}{6}x$ people died

$$\text{People remained} = x - \frac{1}{6}x$$
$$= \frac{5}{6}x \text{ people}$$
$$\frac{1}{3} \text{ of remainder} = \frac{1}{3} \times \left(\frac{5}{6}x\right)$$
$$= \frac{5}{18}x \text{ people}$$
$$\text{People remained} = \frac{5}{6}x - \frac{5}{18}$$
$$= \frac{5}{9}x \text{ People}$$
$$\text{Therefore,} \quad \frac{5}{9}x \text{ People} = 840$$
$$x = 1512 \text{ people}$$

$\therefore$ People left the village is:

$$\frac{5}{18} \times 1512 = 420 \text{ People.}$$

(b) Given the LCM $= 90$ and HCF $= 6$

Then,

$$\text{LCM} = 2 \times 2 \times 3 \times 5 = 90$$
$$\text{and HCF} = 2 \times 3 = 6$$

The first number is $2 \times 3 \times 3 \times 5 = 18$

The second number is $2 \times 3 \times 5 = 30$

$\therefore$ The two numbers are 18 and 30

2\. (a) $5^{3x-4} - 25 = 3100$

$$5^{3x-4} = 3100 + 25 = 3125$$
$$5^{3x-4} = 5^5$$
$$\text{Therefore, } 3x - 4 = 5$$
$$3x = 4 + 5 = 9$$
$$\therefore x = 3$$

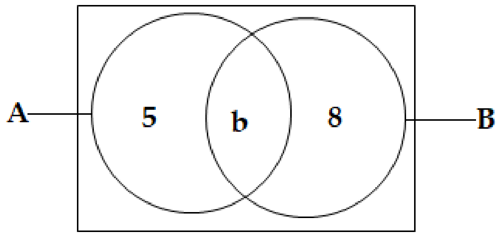
2 (b) $\log\left(\frac{35}{8}\right) + 4\log 2 - \log 2 - \log 7$

$$= \log\left(\frac{35}{8}\right) + 3\log 2 - \log 7$$
$$= \log 35 - \log 8 + 3\log 2 - \log 7$$
$$= \log 35 - \log 2^3 + 3\log 2 - \log 7$$
$$= \log 35 - 3\log 2 + 3\log 2 - \log 7$$
$$= \log 35 - \log 7$$
$$= \log\left(\frac{35}{7}\right) = \log 5$$
$$\therefore \log\left(\frac{35}{8}\right) + 4\log 2 - \log 2 - \log 7 = \log 5$$

(c) Given the expression $\frac{\sqrt{x}+\sqrt{x^3}}{1+\sqrt{x}}$

$$\frac{\sqrt{x}+\sqrt{x^3}}{1+\sqrt{x}} = \frac{\sqrt{x}+\sqrt{x^3}}{1+\sqrt{x}} \times \frac{\sqrt{x}-\sqrt{x^3}}{\sqrt{x}-\sqrt{x^3}}$$
$$= \frac{(\sqrt{x}+\sqrt{x^3})(\sqrt{x}-\sqrt{x^3})}{(1+\sqrt{x})(\sqrt{x}+\sqrt{x^3})}$$
$$= \frac{(\sqrt{x})(\sqrt{x}) - (\sqrt{x^3})(\sqrt{x^3})}{x - x^2 + \sqrt{x} + \sqrt{x} + \sqrt{x^3}}$$
$$= \frac{x - x^3}{x - x^2 + \sqrt{x} + \sqrt{x} + \sqrt{x^3}}$$
$$\therefore \frac{\sqrt{x}+\sqrt{x^3}}{1+\sqrt{x}} = \frac{x - x^3}{x - x^2 + \sqrt{x} + \sqrt{x} + \sqrt{x^3}}$$

3 (a) (i) The Venn Diagram

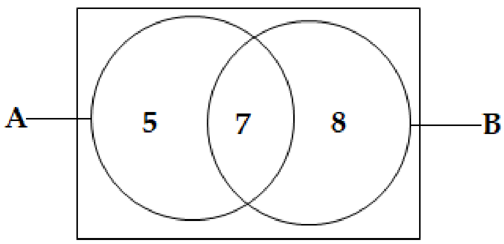


$\cap(\mathrm{A} \cup \mathrm{B}) = 5 + \mathrm{b} + 8 = 20$

$$\mathrm{b} + 13 = 20$$

$$\mathrm{b} = 7$$

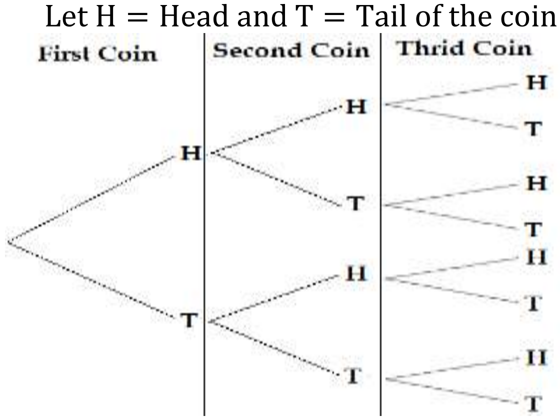
Ther



(ii) $\therefore \cap(A \cap B) = 7$

(b) Solving by a Tree Diagram

Let H = Head and T = Tail of the coin



The Possibility Set is
{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}

Therefore, $\cap(s) = 8$

(i) Getting 2 heads, $\cap(E) = 3$

Then, the probability is $\frac{3}{8}$

(ii) At least 2 heads, $\cap(E) = 4$

Then, the probability is $\frac{4}{8} = \frac{1}{2}$.

4 (a) The equation of the position vectors is:

$$\mathrm{R}(13,4) = \mathrm{a}\{\mathrm{p}(2,2)\} + \mathrm{b}\{\mathrm{q}(3,2)\}$$

$$(13,4) = a(2,2) + b(3,2)$$

$$(13,4) = (2a, 2a) + (3b, 2b)$$

Therefore,

$$2a + 3b = 13 \dots\dots (i)$$

$$2a + 2b = 4 \dots\dots\dots (ii)$$

Solving (i) and (ii) above,

$a = -7$ amd $b = 9$

(b) The point $(-1,7)$ satisfies the equation:

$y = ax + b$

Therefore,

$$7 = -a + b \dots\dots\dots (i)$$

The points of l_2 are $(0.3,0)$ and $(0,-1.5)$

Or in fraction are $\left(\frac{3}{10}, 0\right)$ and $(0, -\frac{3}{2})$

The slope of this line is $\frac{0+\frac{3}{2}}{\frac{3}{10}-0} = \frac{3}{2} \div \frac{3}{10}$

$$= \frac{3}{2} \times \frac{10}{3} = \frac{10}{2} = 5$$

$\therefore$ It's slope is 5.

Therefore, the slope of the $l_1 = -\frac{1}{5} = -0.2$, which is the value of a.

From $7 = -a + b$,

$$7 = -\left(-\frac{1}{5}\right) + b$$

$$7 = \frac{1}{5} + b$$

$$35 = 1 + 5b$$

$$5b = 35 - 1 = 34$$

$$b = \frac{34}{5} = 6.8$$

$$\therefore (a, b) = \left(-\frac{1}{5}, \frac{34}{5}\right) = (-0.5, 6.8)$$

5 (a) Note: the perimeter of the square is same as the circumference of a semi circle.

But perimeter is $4 \times 27 = 108\ cm$

Therefore,

$108 = \frac{\pi d}{2} + d$ (d is the diameter of the circle)

$216 = \pi d + 2d$

$d(2 + \pi) = 216$

$d = \frac{216}{2+\pi} = \frac{216}{2+\frac{22}{7}} = \frac{7\times 216}{14+22}$

$d = \frac{1512}{36} = 42$

But $d = 2r$

$\therefore 2r = 42$ and $r = 21$

It's area is $\pi r^2 = \frac{22}{7} \times 21 \times 21$

$= 1386\ cm^2$

(b) A quadrilateral is a four sided figure

Formula for the length of a side is:

$s = 2r \sin\left(\frac{180}{n}\right)$

$s = 2(10) \sin\left(\frac{180}{4}\right)°$

$= 20 \sin 45°$

$= 20 \times 0.7071$

$= 14.14$ cm

$\therefore$ The length of the side is 14.14 cm

6 (a) For the information given,

$z_1 = \frac{kx}{y}$,

x increased by 12%

$i.e.\quad x + 12\%$ of $x = 1.12x$

Also y decreased by 20%,

$i.e.\ y - 20\%$ of $y = 0.8y$.

Then, the new quantity of z is

$z_2 = \frac{1.12kx}{0.8y}$

Then change in z is

$z_2 - z_1$

$= \frac{1.12kx}{0.8y} - \frac{kx}{y}$,

$= \frac{kx}{y}\left(\frac{1.12}{0.8} - 1\right)$

$= \frac{kx}{y}(0.4)$

6 (a) but $\frac{kx}{y} = z_1$

therefore, the coefficient of z_1, 0.4 is the decimal increase in z.

This increase in percentage form is

$0.4 \times 100\% = 40\%$.

(b) If $1\ Pound = 112\ Ksh$, then

$120\ Pound = 120 \times 112\ Ksh$

$= 13440\ Ksh$

For accommodation $= Ksh\ 1000$

Remainder $= (13440 - 1000)$

$= 12440$

$\frac{1}{4} \times 12440 = Ksh\ 3110$ (For transport)

$12440 - 3110 = Ksh\ 9330$

But, $1\ Pound = 112$, therefore,

$Ksh\ 9330 = \frac{9330}{112}\ Pound$

$= 83.3\ Pound$

$\therefore$ The amount remained after transport and accommodation

$= 83.3\ Pound$

7 (a) Let $x = Boys$ and $y = Girls$

$x + y = 630 \ldots\ldots (i)$

$x : y = 3 : 2$

$\frac{x}{y} = \frac{3}{2}$

$y = \frac{2}{3}x \ldots\ldots (ii)$

Solve (i) and (ii)

$(x, y) = (378,252)$

Now total students after the admission of 90 students is $630 + 90 = 720$ students.

7

(a) Let $b = Boys$ and $g = Girls$

$$b + g = 720 \ldots\ldots (i)$$

$$b : g = 7 : 5$$

$$\frac{b}{g} = \frac{7}{5}$$

$$g = \frac{5}{7}b \ldots\ldots (ii)$$

Solve (i) and (ii)

$(b, g) = (420{,}300)$

∴ New admitted boys $= 420 - 378 = 42$

(b) (i) Required: Gross Profit and Net Profit.

From,

Gross profit = Sales less Cost of goods sold

Where by,

Cost of Goods Sold=COGAS- Closing Stock

But,

COGAS = Opening Stock + Purchases

$= 80{,}000 + 250{,}000$

$= 330{,}000$

COGS = COGAS – Closing Stock

$= 330{,}000 - 50000$

COGS $= 280{,}000$

Gross profit = Sales – COGS

$= 380{,}000 - 280{,}000 = 100{,}000$

∴ Gross profit $= 100{,}000 =$

(ii) Total Expenses

$=$ Electricity $+$ Discount Allowed

$= 40{,}000 + 20{,}000 = 60{,}000$

Net Profit = Gross Profit – Total Expenses

$= 100{,}000 - 60{,}000 = 40{,}000$

∴ Net Profit = =40,000

8.

(a) From,

$$A_n = P\left(1 + \frac{r}{100}\right)^n$$

$$A_2 - A_1 = 6384$$

$$P\left(1 + \frac{r}{100}\right)^2 - P\left(1 + \frac{r}{100}\right)^1 = 6384$$

$$P(1 + 0.14)^2 - P(1 + 0.14) = 6384$$

$$1.29969P - 1.14P = 6384$$

P=40000

∴ She invested sh 40,000

(b) $S_n = \frac{N}{2}(2A_1 + (n-1)d)$

$$10000 = \frac{10}{2}\left(2A_1 + (10-1)(-8)\right)$$

$$10000 = 5(2A_1 - 72)$$

$$2000 = 2A_1 - 72$$

$$2000 = 2(A_1 - 36)$$

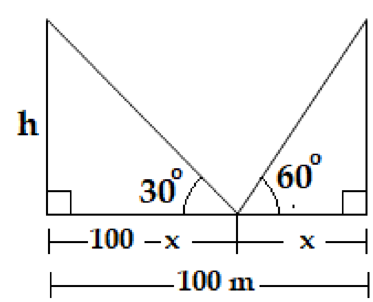
$$1000 = A_1 - 36$$

$$\therefore\ A_1 = 1000 + 36 = 1036$$

6

9.

(a)



$$\tan 30° = \frac{h}{100 - x}$$

$$100 - x = \frac{h}{\tan 30°}$$

$$x = 100 - \frac{h}{\tan 30°}$$

$$x = \frac{100\tan 30° - h}{\tan 30°} \ldots\ldots (i)$$

9. (a) $\tan 60 = \frac{h}{x}$

$$x = \frac{h}{\tan 60°} \ldots\ldots\ldots\ldots.(ii)$$

Substitute (ii)in (ii)

$$\frac{h}{\tan 60°} = \frac{100 \tan 30° - h}{\tan 30°}$$

$$\text{h}\tan 30° = \tan 60° (100 \tan 30° - h)$$

$$= 100 \tan 60° \tan 30° - h \tan 60°$$

$$h \tan 30° + h \tan 60° = 100 \tan 60° \tan 30°$$

$$h(\tan 30° + \tan 60°) = 100 \tan 60° \tan 30°$$

$$h = \frac{100 \tan 60° \tan 30°}{\tan 30° + \tan 60°}$$

$\therefore$ Height is 43.3 m

(b) (i) By Pythagoras theorem:

$$(7x)^2 + (24x)^2 = (150)^2$$

$$49x^2 + 576x^2 = 22500$$

$$625x^2 = 22500$$

$$x^2 = \frac{22500}{625} = 36$$

Therefore, $x = 6$ cm

(ii) Area $= \frac{1}{2}\ base \times Height$

$$= \frac{1}{2} \times (24 \times 6) \times (7 \times 6)$$

$$= 302$$

$\therefore$ It'sarea is 302 cm^2

10. (a) Let $y =$ Total number of pupils

Therefore, available cards $= y(y-1) = 90$

Not: each pupil will have $(y-1)Cards$.

Then, $y(y-1) = 90$

$$y^2 - y = 90$$

$$y = -9\ or\ 10$$

$\therefore$ There are 10 pupils

10 (b) Let the two parts be x and y and x be the large part

Then, $x^2 + y^2 = 20 \ldots\ldots.(i)$

And, $x^2 = 8y \ldots\ldots\ldots\ldots\ldots(ii)$

Substitute (ii) in (i)

$8y + y^2 = 20$

$$y^2 + 8y - 20 = 0$$

$y = -10\ or\ 2$

Using a positive $y = 2$,

$$x^2 = 8y$$

$= 8 \times 2$

$= 16$

$x = \pm 4$

$\therefore$ The positive numbers are 2 and 4

SECTION B

11 (a) (i) **The Frequency Distribution Table**

Marks	X	f	fx
5 - 14	9.5	3	28.5
15 - 24	19.5	7	136.5
25 - 34	29.5	12	354
35 - 44	39.5	20	790
45 - 54	49.5	30	1485
55 - 64	59.5	15	892.5
65 - 74	69.5	8	556
75 - 84	79.5	3	238.5
85 - 94	89.5	2	176
		∑f=100	∑fx=4660

(ii) Median Class is $45 - 54$

(iii) Mean = $\frac{\sum fx}{\sum f} = \frac{4660}{100} = 46.6$

(b) $\overline{AD}^2 = \overline{AB} \times \overline{AP}$

Since D is the mid point of $\overline{AC}$

$\overline{AD} = \frac{1}{2}\overline{AC} = \frac{1}{2}\overline{AB}$

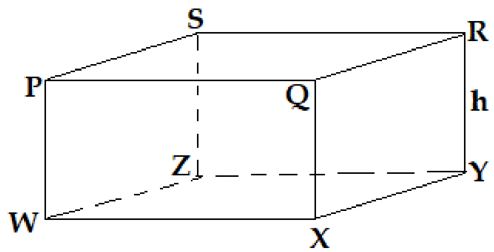
$\overline{AD}^2 = \frac{1}{4}\overline{AB}^2$

$\frac{1}{4}\overline{AB}^2 = AB \times AP$

$$\overline{AB}^2 = 4AB \times AP$$

$AB = 4AP$. Hence proved

12 (a)



(i) Required is $\overline{WX}, \overline{XY}$ and $\overline{RY}$

Area of plane $WXYZ = (WX) \times (XY) = 450$

Area of plane $RSZY = (RY) \times (WX) = 600$

Area of plane $QRYX = (XY) \times (RY) = 300$

Then, $\frac{(RY)\times(WX)}{(RY)\times(XY)} = \frac{600}{300}$

$$\frac{WX}{XY} = \frac{600}{300} = 2$$

$$\therefore WX = 2XY \ldots\ldots\ldots..(i)$$

$$\text{Also, } \frac{(RY)\times(WX)}{(XY)\times(WX)} = \frac{600}{450}$$

$$\frac{RY}{XY} = \frac{600}{450} = \frac{4}{3}$$

$$\therefore 3RY = 4XY \ldots\ldots\ldots(ii)$$

$$\frac{(XY)\times(WX)}{(XY)\times(RY)} = \frac{450}{300}$$

$$\therefore \frac{WX}{RY} = \frac{450}{300} = \frac{3}{2}$$

$$RY = \frac{2WX}{3}$$

From,

$$(RY) \times (WX) = 600$$

$$\therefore RY = \frac{600}{WX}$$

Then, $RY = \frac{2WX}{3} = \frac{600}{WX}$

$$2(WX)^2 = 600 \times 3$$

$$(WX)^2 = \frac{1800}{2}$$

12 (a) $(WX)^2 = 900$

$$\therefore WX = 30 \text{ cm}$$

But $= RY = \frac{600}{WX} = \frac{600}{30} = 20$ cm

From $(XY) \times (RY) = 300$

$XY = \frac{300}{RY} = \frac{300}{20} = 15$ cm

$\therefore$ Dimensions are 15 cm, 20 cm and 30 cm

(ii) Total surface area is 2(600+300)

$= 1800 \text{ cm}^2$

(iii) Volume = Lenght × Width × Height

$= 15 \times 20 \times 30$

$$= 9000\ cm^3$$

(b) The central angle subtended by the

Earths arc is $60 + 60 = 120°$

Distance is $\theta 60 \cos 75°$ nautical miles

(Along small circle)

$$= 120 \times 60 \cos 75$$

$$= 7200 \times 0.2588$$

$$= 18630.5\ nauticalmiles$$

13. (a) Let x = shirt cost and y =trouser cost

$5x + 3y = 1750$

$3x + y = 850$

$$\begin{pmatrix} 5 & 3 \\ 3 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1750 \\ 850 \end{pmatrix}$$

Determinant= $(5 \times 1) - (3 \times 3)$

$$= 5 - 9$$

$$= 4$$

Inverse matrix is $\begin{pmatrix} -\frac{1}{4} & \frac{3}{4} \\ \frac{3}{4} & -\frac{5}{4} \end{pmatrix}$

13. (a) Inverse matrix is $\begin{pmatrix} -\frac{1}{4} & \frac{3}{4} \\ \frac{3}{4} & -\frac{5}{4} \end{pmatrix}$

$$\begin{pmatrix} 5 & 3 \\ 3 & 1 \end{pmatrix}\begin{pmatrix} -\frac{1}{4} & \frac{3}{4} \\ \frac{3}{4} & -\frac{5}{4} \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -\frac{1}{4} & \frac{3}{4} \\ \frac{3}{4} & -\frac{5}{4} \end{pmatrix}\begin{pmatrix} 1750 \\ 850 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 200 \\ 250 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 200 \\ 250 \end{pmatrix}$$

∴ The shirt and trouser cost are

sh 200 and 250 respectively

(b) For the given transformation matrix,

Image, $\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$

The image of $\mathrm{A}(1,-1) = \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 1 \\ -1 \end{pmatrix}$

$a - b = 1 \ldots\ldots\ldots\ldots.(i)$

$c - d = 1 \ldots\ldots\ldots\ldots.(ii)$

The image of $\mathrm{C}(3,-2) = \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 3 \\ -2 \end{pmatrix}$

$3a - 2b = 3 \ \ldots\ldots\ldots\ldots.(iii)$

$3c - 2d = 4 \ \ldots\ldots\ldots\ldots.(iv)$

Solving $(i), (ii), (iii)$ and (iv)

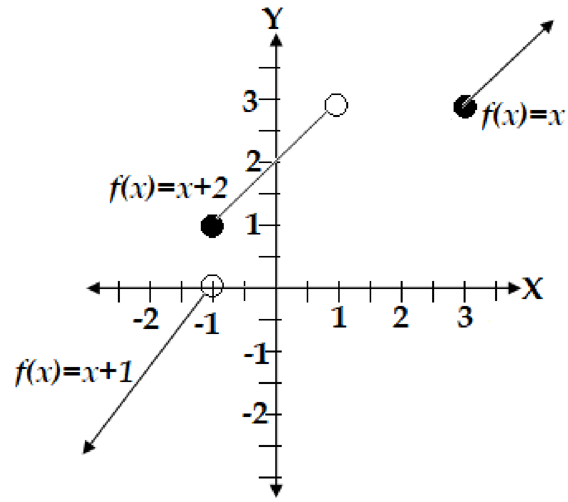
Gives $a = 1, b = 0, c = 2$ and $d = 1$

Then the matrix is $\begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}$

The image of $B(1,-4) = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ -4 \end{pmatrix}$

∴ The image is $(1,-2)$

14 (a) (i) The graph of $f(x)$



(ii) Domain is the set of all real numbers

except $1 \le x < 3$

Range is the set of all real numbers except

$0 \le y < 1$

(iii) The function is one to one because

domain

gives the unique value of range.

(b) Let $A = x$ and $\mathrm{B} = y$

Then the inequalities are:

$\frac{3}{2}x + 3y \le 80$

$3x + 6y \le 160$

$2x + y \le 70$

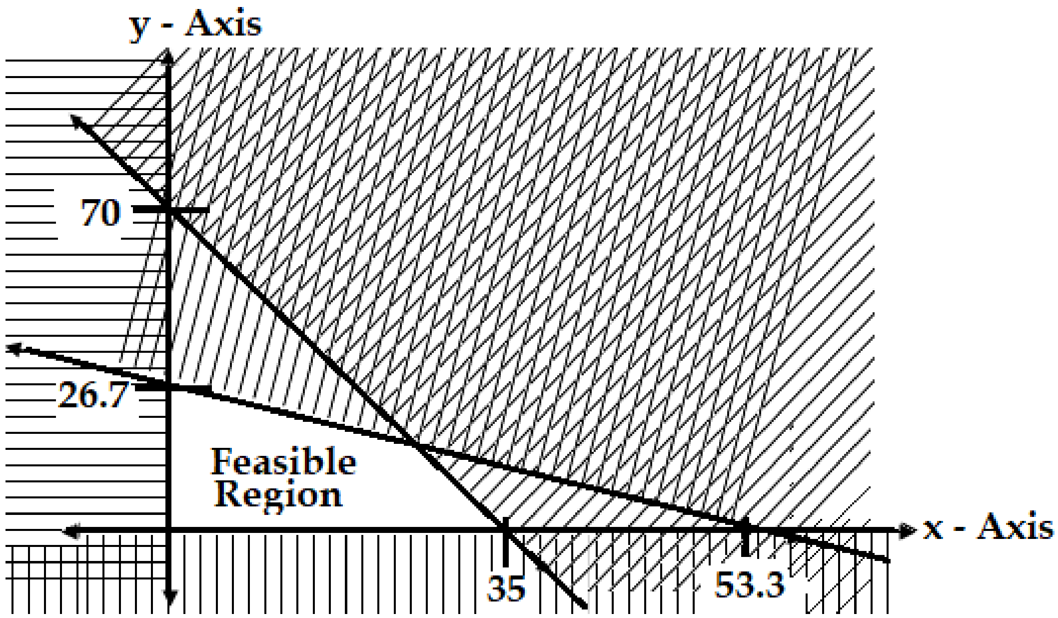
$x \ge 0$

$y \ge 0$

To maximize $f(x, y) + 1000x + 800y$

14 (b)

The Graphs of the Inequalities



Therefore, 28 products and 12 products should be produced to make a maximum profit of 37,600 per week

Corner Points	$f(x,y) = 1000x + 800y$	Value
(0,0)	1000(0) + 800(0)	0
(0,27)	1000(0) + 800(27)	21600
(35,0)	1000(35) + 800(0)	35000
(28,12)	1000(28) + 800(12)	37000